Slope, Linear Equations and Linear Functions (Day 1) Section 13.4. – 13.5 in MATH 0031

Class Strategy:

Instructor Led Discussion: Introduce the concept of average rate of change, including delta notation and attention to unit measure. Students do Activity 1.5 from the supplementary materials.

Discovery Learning Activity: Students do Activity 1.6(Depreciation) – Mathematics in Action – Algebraic, Graphic, & Trigonometric Problems Solving – Second Edition - The Consortium For Foundation Mathematics

You have decided to buy a new Honda Accord LX, but you are concerned about the value of the car depreciating over time. You search the internet and obtain the following information at www. Intellichoice.com:

2004 Accord LX

- Suggested retail price $18,790
- Depreciation per year $1,385 (assume constant)

1.a. Complete the following table in which *V* represents the value of the car after *n* years of ownership.

n, Years	*v*, Value in Dollars
0	
1	
2	
3	
5	
8	

b. Is the value of the car a function of the number of years of ownership? Explain.

c. What is the input? What is the output?

2.a. Select two ordered pairs of the form (*n,v)* from the table in problem 1 and determine the average rate of change.

b. What are the units of measure of the average rate of change?

c. What is the practical meaning of the sign of the average rate of change?

d. Select two different ordered pairs and compute the average rate of change.

e. Select two ordered pairs not used in parts a or d, and compute the average rate of change.

f. Using the results in parts a, d, and e, what can you infer about the average rate of change over any interval of time?

If the computation of the average rate of change using any two ordered pairs yields the same result, the average rate of change is said to be constant.

Definition

Any function in which the average rate of change, $\frac{change\ in\ output}{change\ in\ input}$, is constant is called a **linear function**.

3.a. Is the value, *v*, of the car a linear function of the number of years, *n*, of ownership? Explain using the definition of linear function.

b. Is this function increasing, decreasing, or constant?

4. Replace with activity 1.6 from supplementary materials.

The graph of a linear function is a non vertical line. The constant average rate of change is called the slope of the line and is denoted by the letter *m.*

Definition

If x represents the input variable and y represents the output variable, then the slope m is given by

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1},\ where\ \ x_1 \neq x_2.$$

5.a. What is the slope of the line graphed in problem 4

b. What is the relationship between the slope of the line and the average rate of change?

c. What is the practical meaning of the slope in this situation?

Vertical Intercept

Definition
The vertical intercept is the point where the graph crosses, or intercepts, the vertical axis. The input value of a vertical intercept is always zero. If the output variable is represented by *y*, the vertical intercept is referred to as the *y*-intercept.

6.a. Using the table or data in problem 1or the graph in problem 4, determine the vertical intercept (*V*-intercept).

b. What is the practical meaning of the vertical intercept in this situation?

7a. Review how you determined the value, *v*, of the car in problem 1 for a given number of years, *n*, of ownership. Write an equation for v in terms of *n.*

8. Recall that the slope of your line is $m = -1385$ and the vertical intercept is $(0,18790)$. How is this information contained in the equation of the line you determined in problem 7a?

Definition

The coordinates of all points (x, y) on the line with slope *m* and vertical intercept $(0, b)$ satisfy the equation

$y = mx + b$ *or* $y = b + mx$

This is called the **slope – intercept form** of the equation of line.

Note that the coefficient of *x,* which is *m, is the slope of the line. The constant term, b, is the* y coordinate of the vertical intercept.

Example 1: The slope – intercept form of the equation of the line with slope 3 and vertical intercept $(0, -6)$ is $y = 3x - 6$.

9. Identify the slope and vertical intercept of the line whose equation is given. Write the vertical intercept as an ordered pair.

a. $y = -2x + 5$

b. $s = \frac{3}{4}t + 2$

c. $q = 2 - r$

d. $y = \frac{5}{6} + \frac{x}{3}$

Horizontal Intercepts

Definition

A horizontal intercept of a graph is a point where the graph meets or crosses the horizontal axis. The output value of the horizontal intercept is always zero. If the input variable is represented by *x*, the horizontal intercept is referred to as the *x*-intercept.

Example 2: Consider the equation $y = 2x - 10$.

a. The vertical intercept (*y*-intercept) occurs where the line crosses the vertical axis, i.e., where $x = 0$. Letting $x = 0,\ y = 2(0) - 10$ *or* $y = -10.$ The vertical intercepts is $(0, -10).$

b. The horizontal intercept (*x*-intercept) occurs where the line crosses the horizontal axis, that is , where $y = 0$.

Letting $y = 0$,

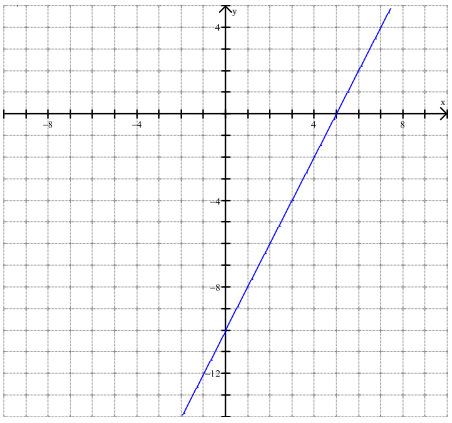
$0 = 2x - 10$

$10 = 2x$

$5 = x$

Therefore, the horizontal intercept is $(5, 0)$

c. You can now sketch the graph of $y = 2x - 10$ by plotting the horizontal and vertical intercepts and connecting the points



10.a. Determine the horizontal intercept (*n-intercept)* of the graph of the car value equation $v = -1385n + 18,790$.

b. What is the practical significance of the horizontal intercept? Include units.

MATH 092
SUMMARY 13.4 , 13.5

Slope, Linear Equations and Linear Functions

1. The average rate of change of a function over a specified input interval is the ratio $\frac{\textit{change in } \_\_\_\_\_}{\textit{change in } \_\_\_\_\_}$. (Use the words "input" and "output" to complete the sentence.)

2. If the input variable is g, the number of gallons of gas used, and the output variable is k, the number of kilometers driven, then the average rate of change of k with respect to g is written in symbols as _____and the unit label is _____per_____.

3. However, if the input variable is *k*, the number of kilometers driven, and the output variable is g, the number of gallons of gas used, then the average rate of change of g with respect to *k* is written in symbols as _____and the unit label is _____per_____

4. A linear function is a function for which the average rate of change between any pair of points remains _____.

5. A linear function is a function whose equation can be written in the form _____, where *m* and *b* are constants.

6. The graph of a linear function is a non vertical _____. The constant rate of change is called the _____and is denoted by the letter _____

7. The formula for computing the slope of the line containing the two points (x_1, y_1) and (x_2, y_2) is _____

8. The slope intercept form of the equation of a line is _____, where *m* is the _____ and *b* is the _____

9. The Vertical intercept (*y*-intercept) $(0,b)$ of a graph is the point where the graph crosses the _____. The horizontal intercept (*x intercept)* $(a,0)$ of a graph is the point where the graph crosses the _____.

10. To find the *b* value of the vertical intercept $(0,b)$, set _____equal to 0 and solve for _____.

11. To find the *a* value of the horizontal intercept $(a,0)$, set _____equal to 0 and solve for _____.

12. If the slope of a linear function is positive, the graph of the function _____(rises, falls) to the right and the function is said to be _____(increasing, decreasing).

13. If the slope of a linear function is negative, the graph of the function _____to the right and the function is said to be _____

14. To scale an axis means to decide how many units each block along the axis equals and then use this to label some tick marks. In the given graph, the scale on the horizontal axis is _____and the scale on the vertical axis is _____.