Average Rate of Change

Functions are used to model the way one quantity changes with respect to another quantity. For instance, how does the distance traveled change as time changes, say from 1 hour to 4 hours? Or how does profit change as the number of items sold changes from 3000 to 4500? To help us study the way quantities change we use a special symbol, Δ (delta). The symbol Δ is used to represent the phrase "change in."

The following table gives *D*, Sam's distance from home in miles, after *t* hours of traveling west on Route 80. Note at time $t = 0$, Sam is 35 miles from his home.

t hours	0	1	3	3.5
D, distance from home (miles)	35	80	200	225

1. How far has Sam traveled in the first hour?

2. We use the symbol ΔD (read "delta D") to represent the change in *D* over a given input interval. We calculate ΔD, the change in distance, by finding the difference between the final distance (D_2) and the initial distance (D_1) and write: $\Delta D = 80 - 35 = 45$ miles

> In general $\Delta D = D_2 - D_1$ or $\Delta D =$ final distance – initial distance
>
> Since *D* is the output, ΔD is called the **change in the output.**

Likewise, the change in time is $\Delta t = 1 - 0 = 1$ hour

> In general $\Delta t = t_2 - t_1$ or $\Delta t =$ final time – initial time
>
> Since *t* is the input, Δt is called the **change in the input.**

3. Find the change in distance from $t = 1$ to $t = 3$ and find the change in time. Be sure to label your answers with the correct units.

$\Delta D =$ ______ – ______ = __________ and $\Delta t =$ _____ – _____ = __________

We can know more about Sam's travel by computing the **ratio** of the change in distance, ΔD, to the change in time, Δt. This is written $\frac{\Delta D}{\Delta t}$ and is read "delta *D* over delta *t*." $\frac{\Delta D}{\Delta t}$ tells us how the distance changed on average over the time interval. We call this ratio $\frac{\Delta D}{\Delta t}$ the average rate of change of distance *D* with respect to time *t*.

> **Definition:** The **average rate of change of a function over a given input interval** is $\frac{\text{change in output}}{\text{change in input}}$. The rate of change is a number that indicates how much and in what direction the output changes when the input changes by one unit.

4. a. Find the average rate of change of Sam's distance from $t=1$ to $t=3$ Use the correct units.

$$\frac{\Delta D}{\Delta t}=\frac{\quad - \quad \text{miles}}{\quad - \quad \text{hours}}=\underline{\qquad\qquad}$$

Practical Meaning: Sam is traveling at an average rate of ________________ during this time interval.

> Note: If you write the correct units in the numerator and in the denominator then your answer label will always be in the form numerator units per 1 denominator unit.
> Example:
> $$\frac{18-6 \text{ dollars}}{6-2 \text{ pounds}}=3 \text{ dollars per pound or } \$3/lb$$

b. Find the average rate of change of Sam's distance from $t=3$ to $t=3.5$ Use the correct units.

$$\frac{\Delta D}{\Delta t}=\frac{\quad - \quad}{\quad - \quad}=\underline{\qquad\qquad}$$

Practical Meaning: Sam is traveling at an average rate of ________________ during this time interval.

5 ~~6.~~ The following table shows Eric's weight over a five week diet program.

Weeks, w	0	1	2	3	4	5
Weight, y (pounds)	190	184	181	184	182	179

a. Find the average rate of change of Eric's weight over the first three weeks (i.e. from $w=0$ to $w=3$.

b. What is the significance of the negative sign in your answer to Part a?

c. Find the average rate of change of Eric's weight from week 1 to week 3.

d. Does your answer to Part c mean Eric's weight did not change during that two week period?

SUMMARY:

1. The average rate of change of a function over a specified input interval is the ratio $\dfrac{\text{change in \_\_\_\_\_}}{\text{change in \_\_\_\_\_}}$. (Use the words "input" and "output" to complete the sentence.)

2. If the input variable is g, the number of gallons of gas used, and the output variable is k, the number of kilometers driven, then the average rate of change of k with respect to g is written in symbols as —— and the unit label is ________per________.

3. However, if the input variable is k, the number of kilometers driven, and the output variable is g, the number of gallons of gas used, then the average rate of change of g with respect to k is written in symbols as —— and the unit label is ________per ________.