# Abstracts of Papers
# Presented at
# MathFest 2018
## Denver, CO
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# Contents

# Invited Addresses

## Earle Raymond Hedrick Lecture Series

**Gigliola Staffilani** Massachusetts Institute of Technology

**Nonlinear Dispersive Equations and the Beautiful Mathematics That Comes with Them**

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**Lecture 1: Thursday, August 2, 11:00–11:50 AM, Plaza Ballroom A, B, & C, Plaza Building**
**Lecture 2: Friday, August 3, 10:30–11:20 AM, Plaza Ballroom A, B, & C, Plaza Building**
**Lecture 3: Saturday, August 4, 10:00–10:50 AM, Plaza Ballroom A, B, & C, Plaza Building**

In these lectures I will give an overview of the rich mathematical structures that characterize the wave solutions of some of the most important nonlinear partial differential equations, such as the Schrödinger equation. In doing so I will illustrate how beautiful pieces of mathematics, developed using different tools, not just coming from analysis, have been generated over the years in order to answer some of the most fundamental questions for these equations, such as existence and uniqueness of solutions for example. Along the way I will formulate open questions and possible new directions of investigation.

## AMS-MAA Joint Invited Address

### Thursday, August 2, 10:00–10:50 AM, Plaza Ballroom A, B, & C, Plaza Building

**Arlie Petters** Duke University

**Gravity's Action on Light: A Mathematical Journey**

The gravitational fields of stars, black holes, and galaxies act on light propagating near them, casting magnification patterns in space. Such optical phenomena have wide-ranging physical applications, including detecting extrasolar planets and testing for a fifth dimension of the universe. Assuming no background in astrophysics or cosmology, this talk will take you on a mathematical journey unveiling the intriguing properties of these beautiful magnification patterns.

## MAA Invited Address

### Friday, August 3, 11:30–9:50 AM, Plaza Ballroom A, B, & C, Plaza Building

**Eugenia Cheng** School of the Art Institute of Chicago

**Inclusion-exclusion in Mathematics: Who Stays in, Who Falls out, Why It Happens, and What We Should Do About It**

The question of why women are under-represented in mathematics is complex and there are no simple answers, only many contributing factors. I will focus on character traits, and argue that if we focus on this rather than gender we can have a more productive and less divisive conversation. To try and focus on characters rather than genders I will introduce gender-neutral character adjectives "ingressive" and "congressive" to replace masculine and feminine. I will share my experience of teaching congressive abstract mathematics to art students, in a congressive way, and the possible effects this could have for everyone in mathematics, not just women. I will present the field of Category Theory as a particularly congressive subject area, accessible to bright high school students, and contrast it with the types of math that are often used to push or stimulate those students. No prior knowledge will be needed.

**Joseph Teran** University of California Los Angeles

**Snow Business: Scientific Computing in the Movies and Beyond**

New applications of scientific computing for solid and fluid mechanics problems include simulation of virtual materials in movie visual effects and virtual surgery. Both disciplines demand physically realistic dynamics for materials like water, smoke, fire, and soft tissues. New algorithms are required for each area. Teran will speak about the simulation techniques required in these fields and will share some recent results including: simulated surgical repair of biomechanical soft tissues; extreme deformation of elastic objects with contact; high resolution incompressible flow; and clothing and hair dynamics. He will also discuss a new algorithm used for simulating the dynamics of snow in Disney's animated feature film, "Frozen" .

**Lisette de Pillis** Harvey Mudd College

**Mathematical Medicine: Modeling Disease and Treatment**

Immune system dynamics have proven to play an increasingly central role in the development of new treatment strategies for immune-related diseases such as type 1 diabetes and certain cancers. The critical importance of the immune system in fighting such diseases has been verified clinically, as well as through mathematical models. Many open questions remain, however, including what may lead to non-uniform patient responses to treatments, and how to optimize and personalize therapy strategies. Mathematical models can help to provide insights into the mechanisms that may be influencing patient outcomes. In this talk, we will present a sampling of mathematical models that help us to simulate immune system interactions, disease dynamics, and treatment approaches that may slow, or even stop, disease progression.

# MAA James R.C. Leitzel Lecture

## Saturday, August 4, 9:00–9:50 AM, Plaza Ballroom A, B, & C, Plaza Building

**Talitha Washington** Howard University and National Science Foundation

**The Relationship between Culture and the Learning of Mathematics**

How do we ensure that our mathematics is culturally inclusive? Why have issues with minority participation not been resolved? Unfortunately, even with our best intentions, our implicit biases impact the mathematics we teach and learn. We all can take an active role to ensure the strength of our future mathematical community, which should also be a reflection of our Nation. I will share how to infuse various cultures in learning mathematics that can better help educate those of diverse backgrounds which will broaden the participation of those doing mathematics.

# AWM-MAA Etta Z. Falconer Lecture

## Friday, August 3, 9:30–10:20 AM, Plaza Ballroom A, B, & C, Plaza Building

**Pamela Gorkin** Bucknell University

**Finding Ellipses**

Ellipses make frequent appearances in our lives: Kepler's laws of planetary motion involve ellipses and a medical procedure involving kidney stones known as lithotripsy uses them as well. We see ellipses in architecture and in President's Park South we find a park called simply "The Ellipse." What properties of the ellipse make it so important? How can we construct an ellipse? We begin with questions like these, providing some unfamiliar answers. Then we study three seemingly unrelated problems in mathematics, chosen from linear algebra, complex analysis, and projective geometry, and we show how the solution to each of these problems relies on finding ellipses.

# MAA Chan Stanek Lecture for Students

## Thursday, August 2, 1:30–2:20 PM, Plaza Ballroom A, B, & C, Plaza Building

**Laura Taalman** James Madison University

**FAIL: A Mathematician's Apology**

The job of being a mathematician primarily consists of long periods of failure punctuated by short bursts of success which later seem to be somewhat obvious...but that's what we love about it! And, as it turns out, 3D printing kind of works the same way. In this talk we'll take a journey through many mathematical and 3D printing failures and try to laugh about it the best we can.

# Pi Mu Epsilon J. Sutherland Frame Lecture

## Wednesday, August 1, 8:00–8:50 PM, Plaza Ballroom A, B, & C, Plaza Building

**Peter Winkler** Dartmouth College

### The Singular Uniformity of Large Random Systems

A random structure could be anything, yet somehow, when that structure is composed of many small parts, it often turns out to be shockingly predictable—at least, in a probabilistic sense. A random graph on a million vertices, for example, has a long list of characteristics each with high probability. In an attempt to understand this phenomenon, we'll take a little tour from zero-one laws to variational principles, contrasting graphs and permutations along the way.

# NAM David Harold Blackwell Lecture

## Friday, August 3, 1:30–2:20 PM, Plaza Ballroom A, B, & C, Plaza Building

**Raegan Higgins** Texas Tech University

### Continuous, Discrete, or Somewhere in Between: An Introduction to Time Scales with Applications

Since Stefan Hilger's landmark paper in 1988, progress has been made in the unification and extension of discrete and continuous analysis. The broad idea is to prove a result once for a dynamic equation where the domain of the unknown function is a time scale T, which is an arbitrary, nonempty, closed subset of the real numbers. In this talk, we will use the exponential function ept to introduce the theory of time scales. Considering a certain second-order linear delay dynamic equation, we establish some sufficient conditions which ensure that every solution oscillates. The obtained results unify the oscillation of second-order delay differential and difference equations. Our interest in delay equations has lead us to study a certain area of mathematical physiology. We are using mathematical models to understand how behavioral disruption of the circadian clock can lead to glucose dysregulation. In this talk, we present some preliminary results.

# Alder Awards

## Alder Awards

### Friday, August 3, 2:30–3:50 PM, Plaza Ballroom, A, B, & C, Plaza Building

**Mohamed Omar** Harvey Mudd College

#### Creativity Amidst Adversity

*2:30–2:50* PM

In times of personal struggle, it is very difficult to think about our work, and we often struggle with the question "How do I move forward?" This talk explores the various ways in which I have coped, managed, and even shined as a life-long student of mathematics amidst my emotional and physical adversities. Two major themes will emerge. One is how my limitations forced me to be creative in the classroom, and how fruitful that was for my growth as a teacher. Another is my renewed perspective on students' struggles in mathematics.

**David Clark** Grand Valley State University

#### Way to Fail!

*3:00–3:20* PM

Struggling and failing is part of learning – "productive failure". We know this. We say this to students. Do we really walk this walk when we teach? If our goal is to get students actively engaged in their own learning, we have to be ready for them to struggle and even fail when they encounter new ideas. To help students learn from this, we must build meaningful support for productive failure into our classes from the ground up. I'll share some ways that I (try to) encourage my students to struggle, fail, and then learn from it – not just superficially, but as key parts of their learning experiences. I'll emphasize this by talking about some ways that I've failed myself, and still lived to talk about it.

**Chad Awtrey** Elon University

#### Mathematics by Design

*3:30–3:50* PM

Mathematics programs "should be designed so that all students come to see mathematics as an engaging field, rich in beauty, with powerful applications to other subjects and contemporary open questions." This is my favorite line from the 2015 Curriculum Guide to Majors in the Mathematical Sciences, which is itself a project of MAA's Committee on the Undergraduate Program in Mathematics (CUPM). But how can we accomplish such a design? What can be done at the programatic level? at the course level? on a daily basis? In this talk, I will discuss a possible framework for answering such questions, called "backward design". Along the way, we will explore (1) findings from the National Research Council concerning how people learn; (2) why and how to implement active learning strategies; and (3) how to use the cognitive and content goals listed in the CUPM's Curriculum Guide as a starting point.

# Invited Paper Sessions

## The MAA Instructional Practices Guide in Action

### Thursday, August 2, 3:00–5:30 PM, Plaza Ballroom E, Plaza Building

**Organizers: Martha Abell** Georgia Southern University
**Carolyn Yackel** Mercer University

The goal of the session is to bring the new MAA Instructional Practices (IP) Guide to life for the mathematical community. Talks will demonstrate how members of the community are using the IP Guide in their classroom practice or for professional development.

**Hortensia Soto** University of Northern Colorado

#### Professional Development for Collegiate Instructors with the MAA Instructional Practices Guide

*3:00–3:20* PM

There is a long history of providing professional development (PD) for K-12 mathematics teachers and for the past decade we have begun to see the implementation of PD opportunities for mathematics teaching assistants. Similar opportunities are rare for collegiate mathematics professors. In this presentation, I will share how I am incorporating ideas from the IP Guide with instructors who teach at two- and four-year colleges as part of a 5-year grant. I intend to highlight how the structure of the IP Guide facilitates such PD.

**Gulden Karakok** University of Northern Colorado

#### Graduate Teaching Assistant Development via the MAA Instructional Practices Guide

*3:30–3:50* PM

The MAA's study, "The Characteristics of Successful College Calculus Programs" reinforces the importance of the effective training of graduate student teaching assistants (GTAs), which is consistent with prior research. In particular, some research studies pointed to GTAs' beginning level of mathematics knowledge for teaching, and their possibly novice beliefs about teaching and learning mathematics. Given that lower level mathematics courses (e.g., College Algebra) and Calculus recitations are often taught by GTAs and such courses are stated to be gatekeepers to STEM, how such courses are taught needs careful consideration. In this presentation, I will present a course design that utilized the MAA Instructional Practices (IP) Guide and has potentials to be a professional development for GTAs. Throughout this required one credit course, the ten GTAs read, discussed and implemented ideas from the IP guide, which provides many practical ideas that GTAs can "quickly" employ in their courses. Furthermore, vignettes and practical tips in this document helped to facilitate rich discussions on teaching mathematics among GTAs. Additional activities and teaching frameworks incorporated to enrich the discussions will also be shared at the presentation.

**Angie Hodge** Northern Arizona University

#### Developing Persistence in Problem Solving in Relation to the MAA Instructional Practices Guide

*4:00–4:20* PM

In order to implement many of the suggestions provided in the MAA Instructional Practices Guide, an instructor has to teach students how to be persistent when faced with mathematical tasks. Due to fixed mindsets that many students have about mathematics, this can be challenging. Ideas will be provided in this session on how to foster a learning atmosphere that encourages mathematical persistence in alignment with the IP Guide. Techniques for encouraging persistence in mathematics will be shared for a range of students such as pre-service teachers and STEM majors.

**April D. Strom** Scottsdale Community College

#### Paired Board Work is Definitely Not Bored Work

*4:30–4:50* PM

Increasing student engagement in the classroom is essential for increasing students' ability to problem solve, construct mathematical arguments, and critique the reasoning of others. An effective active learning strategy, called paired board work, provides students the opportunity to learn from their peers in a structured environment. Students enjoy having the opportunity to gain perspectives from other students on mathematical tasks and discover other methods for completing these tasks. This session will engage the audience in experiencing a sample enactment of a paired board work exercise.

**James A. Mendoza Álvarez** The University of Texas at Arlington

**Five Essential Elements for Cooperative Learning described in the MAA Instructional Practices Guide**
*5:00–5:20* PM

Increasingly, research findings indicate that enhanced learning in the classroom occurs when students are engaged or active in their learning. Using cooperative learning strategies can be one way of fostering student engagement. As described in the MAA Instructional Practices Guide, there are five basic elements essential for successful cooperative learning. I will present examples that build upon cooperative learning strategies presented in the guide and highlight how these examples incorporate these basic elements. I will also discuss ways that instructors can tweak their teaching practice to incorporate these strategies aimed at increasing student engagement in the classroom.

# Bridging Network Science and Graph Theory

## Thursday, August 2, 1:30–4:20 PM, Grand Ballroom II, Tower Building

**Organizer: Ralucca Gera** Naval Postgraduate School

The current session aims at bringing together researchers from different areas to learn or apply their knowledge to network science. While the foundations of Network science are in graph theory, the discipline evolved to include sociologists, computer scientist and others that are interested in understanding and analyzing social networks, technological network, biological networks and networks of information. The network science field bloomed as big data emerged, yet mathematicians are a minority at these conferences. The types of contributions for this session are either state-of-the art overviews of network science research topics, or newly developed theory/applications in network science that is of interest to the mathematical community.

**Ralucca Gera** Naval Postgraduate School

**Teaching Graph Theory and Network Science**
*1:30–1:50* PM

We illustrate how the Naval Postgraduate School's math department augmented the teaching of graph theory by adding a network science course. We present an overview of some classical topics in graph theory, and their transition and extension in the network science world. We indicate some early examples that motivate the study of network science by demonstrating a range of applications that can be analyzed with network concepts and tools.

**Jon Roginski** United States Military Academy

**Teaching Network Science at Different Academic Levels**
*2:00–2:20* PM

Productive learning environments strike a balance between student motivation and the necessary learning outcomes associated with a particular course. We explore ways to achieve such a balance and discusses some of the subtle benefits that spring from that balance. We present course designs that foster student motivation and encourages independent learning, where students began to see coursework, concepts, and feedback as productive and globally meaningful rather than corrective and locally meaningful. We overview teaching network science to undergraduates, graduate students, short courses and elementary school teachers.

**Ruriko Yoshida** Naval Postgraduate School

**Tropical Principal Component Analysis and its Application to Phylogenetics**
*3:00–3:20* PM

Principal component analysis is a widely-used method for the dimensionality reduction of a given data set in a high-dimensional Euclidean space. Here we define and analyze two analogues of principal component analysis in the setting of tropical geometry. In one approach, we study the Stiefel tropical linear space of fixed dimension closest to the data points in the tropical projective torus; in the other approach, we consider the tropical polytope with a fixed number of vertices closest to the data points. We then give approximate algorithms for both approaches and apply them to phylogenetics, testing the methods on simulated phylogenetic data and on an empirical dataset of Apicomplexa genomes. This is joint work with Lean Zhang UC Berkeley and Xu Zhang at u of Kentucky.

**Karl Schmitt** Valparaiso University

#### Using Machine Learning to Classify and Characterize Networks

*3:30–3:50* PM

Networks are often labeled according to the underlying phenomena that they represent, such as re-tweets, protein interactions, or web page links. This research seeks to use machine learning techniques to gain a better understanding of the categories of networks on the Network Repository (www.networkrepository.com) and then classify unlabeled networks into categories that make sense. It is generally believed that networks from different categories have inherently unique network characteristics. This research provides conclusive evidence to validate this belief by presenting the results of global network clustering and classification into common categories using machine learning algorithms. The machine learning techniques of Decisions Trees, Random Forests, Linear Support Vector Classification and Gaussian Naive Bayes were applied to a 14-feature "identifying vector" for each graph. During cross-validation, the best technique, Gaussian Naive Bayes, achieved an accuracy of 92.8%. After training the machine learning algorithm it was applied to a collection of initially unlabeled graphs from the Network Repository. Results were then manually checked by determining (when possible) original sources for these graphs. Finally, we examined the accuracy of our results and discussed how future researchers can make use of this process. This is joint work with Ryan Rossi (Adobe Research), Nesreen Ahmed (Intel Labs), Sucheta Soundarajan (Syracuse University), James Canning (SUNY Geneseo), Emma Ingram (University of Alabama), Adriana Ortiz (University of Puerto Rico), and Sammantha Nowak-Wolff (Valparaiso University).

**Pivithuru Wijegunawardana**

#### Seeing Red: Locating People of Interest in Dark Networks

*4:00–4:20* PM

Dark networks, which describe networks with covert entities and connections such as those representing illegal activities, are of great interest to intelligence analysts. However, before studying such a network, one must first collect appropriate network data. Collecting accurate network data in such a setting is a challenging task, as data collectors will make inferences, which may be incorrect, based on available intelligence data, which may itself be misleading. In our work, we consider the problem of how to effectively sample dark networks, in which sampling queries may return incorrect information, with the specific goal of locating people of interest. We present RedLearn and RedLearnRS, two algorithms for crawling dark networks with the goal of maximizing the identification of nodes of interest, given a limited sampling budget. RedLearn assumes that a query on a node can accurately return whether a node represents a person of interest, while RedLearnRS dispenses with that assumption. We consider realistic error scenarios, which describe how individuals in a dark network may attempt to conceal their connections. We evaluate the performance of the algorithms on several real-world networks, including dark networks, as well as various synthetic dark network structures proposed in the criminology literature. Our analysis shows that RedLearn and RedLearnRS meet or outperform other sampling algorithms. This is joint work with Vatsal Ojha, Ralucca Gera, and Sucheta Soundarajan.

## Geometric Ideas and Where to Find Them

### Friday, August 3, 1:30–4:20 PM, Plaza Ballroom D, Plaza Building

**Organizers: Ulrich Daepp** Bucknell University
**Pamela Gorkin** Bucknell University
**Karl Voss** Bucknell University

Results from geometry have long captivated the attention of mathematicians because of the surprising beauty, wide utility, and intriguing proofs behind the results. Geometric concepts are often a thread connecting areas of mathematics as well as a link between mathematics and other fields. In this session, we focus on new ways of looking at geometric theorems as well as applications to various fields of mathematics, including linear algebra, complex analysis, and dynamics.

**Greg Quenell** State University of New York, Plattsburgh

#### String Art and Calculus

*1:30–1:50* PM

Draw lines connecting (1,0) to (0,9), (2,0) to (0,8), (3,0) to (0,7), and so on. A curve starts to appear in the first quadrant. It looks like a branch of a hyperbola, but it isn't. We consider this curve and other curves that show up in a similar situation: we drive two lines of nails into a board and stretch pieces of string between the nails in one line and those in the other. By varying the spacing of the nails or placing the nails along more general paths, we generate a whole gallery of "string art" curves.

**Karl Voss** Bucknell University

**From Benford's Law to Poncelet's Theorem**
*2:00–2:20* PM
Sometimes seemingly unrelated mathematical ideas do in fact have a connection, in this case through geometry. Benford's law describes an unexpected distribution of first digits in many sets of data that we encounter in the world. As counter-intuitive as Benford's Law may seem upon first examination, there is an unanticipated connection to Poncelet's Theorem, a remarkable geometric result. The journey connecting these ideas links statistics, geometry, and analysis.

**Dan Kalman** American University

**Ellipses ...**
*2:30–2:50* PM
This talk surveys interesting problems, contexts, and results I have encountered over the years. They are united by two common themes. Each is closely connected with ideas in the undergrauate math curriculum, and in some way, each focusses on properties of ellipses or ellipsoids. The topics considered will include the "ladder round a corner" problem, the minimum flight-path angle of a satellite, the structural safety of coal mines, and the optimal speed for a cow running in the rain.

**Sarah Greenwald** Appalachian State University

**Geometry of the Earth and Universe**
*3:00–3:20* PM
The quest to understand the shape of our earth and universe began thousands of years ago, when mathematicians and astronomers used mathematical models to try and explain their observations. We'll explore historical and current theories as we focus on various ways that the classification of objects, the sum of the angles in a triangle, and other geometric ideas relate. In the process we will see connections to algebra, abstract algebra, physics, philosophy and art.

**Sephan Garcia** Pomona College

**The Graphic Nature of Gaus sian Periods**
*3:30–3:50* PM
At the age of eighteen, Gauss established the constructibility of the 17-gon, a result that had eluded mathematicians for two millennia. At the heart of his argument was a keen study of certain sums of complex exponentials, known now as Gaussian periods. It turns out that these classical objects, when viewed appropriately, exhibit dazzling array of visual patterns of great complexity and remarkable subtlety.

**Annalisa Crannell** Franklin & Marshall College

**Gaining Perspective on Homographies**
*4:00–4:20* PM
This is a talk about how to look at the world—and at pictures of the world—with geometry. If we have a perspective picture of objects in a plane, we want to answer questions about where in the world the artist was, where in the world the picture was, and just how distorted the image was. To do this, we'll use a function called a homography, a map from a plane to a plane that preserves lines. The most familiar kinds of homographies to mathematicians are affine maps, which preserve parallel lines, but perspective maps (for example, photographs of parallel railroad tracks that appear to intersect on the horizon) are another form of homographies. In this talk, we'll show that in some precise sense, affine maps and perspective maps are the only kinds of homographies. Specifically, if a homography is non-affine, it must be a perspective mapping composed with an isometry. (In essence, if we put the image in the right location, and stand in the correct spot in the world, then the image will appear to line up exactly with the world).

# Modeling Biological Rhythms

## Friday, August 3, 1:30–4:50 PM, Plaza Ballroom E, Plaza Building

**Organizer: David Brown** The Colorado College

Periodic oscillations are a characteristic feature of many living systems. Cells, organs, and whole organisms often exhibit regular clock-like behavior. Examples include circadian rhythms, heartbeats, brain waves, and the synchronization of behaviors across populations. Researchers seek to understand how these oscillations are generated, how they interact with external cues, and how they persist in the presence of noise. Mathematical modeling has proven to be an invaluable tool for investigating biological rhythms. Drawing on the theory of dynamical systems, mathematical biologists have made important contributions to understanding the structure and behavior of biological oscillators. In addition, these systems are a rich source of topics for classroom explorations and student research projects. Speakers in this IPS will illustrate

the breadth of biological questions and mathematical techniques that are used to study the rhythms of life. They will highlight recent advances and open questions.

**Matthew Mizuhara** The College of New Jersey

### Order Emerging from Chaos: The Mathematics of Firefly Synchronization

*1:30–1:50* PM

Tourists flood Amphawa, Thailand every year to witness nature's light show: trees lined with fireflies blinking in near perfect unison. Such collective behaviors arise spontaneously in many biological systems ranging from bird flocks to neuron synapses to bacterial swimmers. In this talk we will study the Kuramoto model, a system capturing synchronous dynamics of random, coupled oscillators. We will explore the role of network connectivity on synchronization, as well as a variety of coherent structures that can arise. This work is in collaboration with Hayato Chiba and Georgi Medvedev.

**Subekshya Bidari** University of Colorado

### Optimizing Flexibility in the Collective Decisions of Honeybees

*2:00–2:20* PM

Honeybees make decisions as a group while searching for a new home site or foraging. The quality of each choice influences the rate at which scout bees recruit others via a waggle dance. In addition, decided bees can influence those with opposing opinions to change their minds via "stop-signals." Most previous experimental studies have assumed bee swarms make decisions in static environments, but most natural environments are dynamic. In such cases, bees should adapt to new evidence as the environment constantly changes. One way of adapting is to abandon one's current opinion and restart the evidence-accumulation and decision process (Seeley et al 2012). Incorporating such individual behavior into a dynamical model leads to a collective decision-making process that discounts previous evidence and weights newer information more strongly. We show that properly tuning this "forgetting" process can improve a swarm's performance on a foraging task in a dynamic environment. Individual forgetfulness allows the group to change its mind, and move to a higher yielding foraging site. Our analysis explores parameter-dependent changes in the foraging yield using bifurcation theory and fast/slow analysis in a mean field version of the collective decision-making model.

**Per Sebastian Skardal** Trinity College

### Patterns of Collective Oscillations: Effects of Modularity and Time-Dela

*2:30–2:50* PM

Synchronization in large ensembles of coupled oscillators is a nonlinear phenomenon of vital importance in applications ranging from cardiac pacemakers to circadian rhythms. The analytical treatment of such systems has a rich history and remains an active area of research. In this talk we will first explore some of these analytical methods, including a recent dimensionality reduction method discovered by Ott and Antonsen. Using this technique we will then analyze the dynamics of two different oscillator systems with important properties that arise in a wide variety of biological models: modular (i.e., community) structure, and time-delayed interactions. In modular systems, when a system is comprised of a small number of modules we uncover dynamical states corresponding to incoherence, global synchronization, as well as another state characterized by complex oscillations, and bistability between these different states. However, when the number of modules is large we uncover a hierarchical path to synchronization, where modules first synchronize within, then different modules synchronize with one another. In time-delayed systems, we characterize a transition between incoherence and global synchronization that becomes subcritical for sufficiently large time-delays, creating a hysteresis loop between these two states with a region of bistability.

**Nora Stack** Colorado School of Mines

### Establishing a Theoretical Framework for Ultradian Forced Desynchrony Protocols

*3:00–3:20* PM

Humans have an average intrinsic circadian period of $\sim$ 24.2 hours but are entrained to a 24 h day by environmental cues such as light, eating, and exercise. Our work is focused on optimal protocol design and data mining for circadian studies. Recently, we have focused on ultradian forced desynchrony (FD) protocols. These protocols are used to assess the intrinsic period of an individual using short light/dark (LD) cycles (e.g., 4 h LD cycles). These cycles are too short to entrain the circadian pacemaker. Therefore, they decouple the individual's circadian and rest/activity cycles. These short LD cycle protocols have been used less widely compared to traditional FD protocols involving long (e.g., 28 h) LD protocols and an optimal design for ultradian FD protocols has not been established. It is cost prohibitive to optimize ultradian FD protocols experimentally. However, we used a theoretical approach to optimize protocol design and quantify the relative error associated with estimates made using ultradian FD protocols under different experimental conditions. Applying a mathematical model of the circadian pacemaker, we simulated the effects of varying light intensity, light/dark cycle duration, and phase onset for a range of intrinsic periods to determine optimal ultradian protocol design as well as an analysis of potential error present in current ultradian data sets. By investigating the properties of these protocols, we are able to recommend optimal features for protocol design and establish error bounds for existing data sets.

**Yangyang Wang** The Ohio State University

**Multiple Time Scale Bursting Dynamics and Complex Bursting Patterns in Respiratory Neuron Models**
*3:30–3:50* PM

Central pattern generators may exhibit behavior, including bursting, involving multiple distinct time scales. Our goal is to understand bursting dynamics in multiple-time-scale systems, motivated by respiratory central pattern generator neurons. We apply geometric singular perturbation theory to explain the mechanisms underlying some interesting forms of bursting dynamics involving multiple forms of activity within each cycle. We consider how many time scales are involved, obtain some non-intuitive results, and identify solution properties that truly require three time scales.

**Heather Zinn Brooks** University of Utah

**Quasicycles in the Stochastic Hybrid Morris-Lecar Neural Model**
*4:00–4:20* PM

Intrinsic noise arising from the stochastic opening and closing of voltage-gated ion channels has been shown experimentally and mathematically to have important effects on a neuron's function. Study of classical neuron models with stochastic ion channels is becoming increasingly important, especially in understanding a cell's ability to produce subthreshold oscillations and to respond to weak periodic stimuli. While it is known that stochastic models can produce oscillations (quasicycles) in parameter regimes where the corresponding deterministic model has only a stable fixed point, little analytical work has been done to explore these connections within the context of channel noise. Using a stochastic hybrid Morris-Lecar (ML) model, we combine a system-size expansion in K+ and a quasi-steady-state approximation in persistent Na+ in order to derive an effective Langevin equation that preserves the low-dimensional (planar) structure of the underlying deterministic ML model. By calculating the corresponding power spectrum, we determine analytically how noise significantly extends the parameter regime in which subthreshold oscillations occur. This work is joint with Paul Bressloff at University of Utah.

**Greg Handy** University of Utah

**Investigation of Calcium Dynamics in Astrocytes via Bifurcation Analysis**
*4:30–4:50* PM

Astrocytes are glial cells in the brain that make up 50% of brain volume, with each one wrapping around thousands of synapses. In the presence of neuronal activity, astrocytes exhibit calcium transients, hinting that these cells may be playing an active role in regulating brain activity. As a first step in understanding these calcium transients, we examine experimental data collected by our collaborators, in which the calcium responses in astrocytes are evoked artificially by brief stimulant applications. Surprisingly, even in this controlled experimental setup, calcium transients exhibit a vast range of amplitudes and durations, with some presenting multiple calcium peaks after one stimulus. In order to better understand this experimentally observed diversity of calcium transients, we develop an idealized differential equations (ODE) model and investigate the underlying structure of steady states and oscillatory attractors (bifurcation diagram). We use this analysis to propose a classification system for the types of calcium transients observed, and make experimentally testable predictions regarding the mechanisms responsible for the variability observed in our collaborators' experimental data.

# Strategies to Synergize Culture in the Learning and Doing of Mathematics

## Saturday, August 4, 1:30–3:20 PM, Plaza Ballroom E, Plaza Building

**Organizer: Talitha Washington** Howard University and the National Science Foundation

How do we embed various cultures into the learning and doing of mathematics? What are the ways that we can enhance the learning of mathematics through culturally-responsive teaching? Mathematics grounded in the African American, Latinx, and Native American traditions as well as other international traditions can stimulate connections and a sense of belonging in the mathematical community. Presenters will provide implementable strategies to synergize culture in the learning and the doing of mathematics. By infusing various cultures into our mathematics, we enhance the learning experience as well as broaden the inclusion of those doing mathematics.

**Bob Megginson** University of Michigan

**Importance of Culture in Indigenous Learning of Mathematics**
*1:30–1:50* PM

This presentation will focus on the importance of culture in the learning and doing of mathematics in particular indigenous groups. Examples will be given from indigenous cultures of the Western Hemisphere and New Zealand.

**Jacqueline Leonard** University of Wyoming

**Using Computer Modeling to Integrate Culture & Mathematics**
*2:00–2:20* PM

Focus on the intersectionality of computer science, 21stcentury skills, and information and communications technology (ICT) to prepare students for the science, technology, engineering, and mathematics (STEM) workforce is a national priority. Research has shown strong evidence that computer modeling and coding can motivate student learning and broaden opportunities in STEM. Likewise, research has shown the benefits of computing as a pathway to develop computational thinking (CT) skills. Computer modeling provides all students with opportunities to learn CT through real-world applications that may be connected seamlessly to students' culture. In this session, the results to a STEM summer camp on computer modeling within the cultural and place-based context of Yellowstone National Park will be shared with participants.

**Edray Goins** Pomona College

**Diary of a Black Mathematician: From Research I to Liberal Arts**
*2:30–2:50* PM

Last year I made the transition from a Research I University to a Liberal Arts College. In this presentation, I'll outline the reasons why I made the switch, give some stories about my first impressions of the differences in institutions, and discuss how my life as an African American mathematician has changed.

**Rochelle Gutiérrez** University of Illinois

**Rehumanizing Mathematics: Should That Be Our Goal?**
*3:00–3:20* PM

For far too long, we have embraced an "equity" standpoint that has been poorly defined (Gutiérrez, 2002) or constantly shifting (NCTM, 2008). It has been difficult to assess progress beyond closing the achievement gap or recruiting more diverse students into the mathematical sciences. Instead, we should rehumanize mathematics, which considers not just access and achievement, but the politics in teaching and mathematics. This approach begins with 1) acknowledging some of the dehumanizing experiences in mathematics for students and teachers and 2) how students could be provided with windows and mirrors onto the world and ways of relating to each other with dignity. As such, we can begin to think differently about student misconceptions, teachers as identity workers, and why it is not just that diverse people need mathematics but mathematics needs diverse people (Gutiérrez, 2002; 2012). In this talk, I present eight dimensions of a rehumanized mathematics experience: participation/positioning; cultures/histories; windows/mirrors; living practice; broadening mathematics; creation; body/emotions; and ownership. Then, I offer ways for mathematicians and mathematics educators to take risks in ensuring those dimensions happens in small and large ways.

# Category Theory for All

## Saturday, August 4, 1:30–4:20 PM, Plaza Ballroom D, Plaza Building

**Organizer: Eugenia Cheng** School of the Art Institute of Chicago

Category theory can be thought of as being "very abstract algebra". It is typically taught at graduate school or in some select cases to advanced undergraduates. In this session we will show ways in which category theory can be taught in a meaningful way to undergraduates and those without particularly aptitude or expertise in math, even high school and middle school students. In the process, we will emphasize important aspects of mathematics that are not to do with solving problems, proving theorems, or getting the right answer, including: making connections between different situations, illuminating deep structures, finding fundamental reasons for things, and improving the clarity of our thinking. The talks will be of interest for general enrichment as well as pedagogy.

**Alissa Crans** Loyola Marymount University

**Making Distinctions: Interpreting the Notion of Sameness**
*1:30–2:05* PM

Walgreens or CVS? Same difference if you're just stopping on the way home for a box of tissues. But certainly not to the employee whose shift starts tonight at 5 pm! As we know, a fundamental problem in mathematics consists of determining whether two given mathematical structures are 'the same'. But what exactly do we mean when we say that two gadgets are the same? Often, we mean "sufficiently the same for our purposes," and that purpose naturally differs from field to field. We will explore mathematical interpretations of being 'the same' by carefully examining the concept of equality and comparing it to weaker notions of sameness. No prior knowledge of category theory will be assumed.

**Sarah Yeakel** University of Maryland

### Social Choice and Functoriality

*2:15–2:50* PM

A town wants to vote on the placement of a statue in a park. Is it possible to find a solution where everyone gets an equal say? The question boils down to a topological problem; what does the park look like? But whether such a vote will result in a happy town is more easily answered by looking at some group theory. The reason this comparison works is because of a piece of category theory that facilitates moving between different mathematical worlds. Functoriality is the esssentially principle of mapping from one world to another while respecting aspects of its mathematical structure. It enables us to study one world via a different one that we might understand better. In this talk we will discuss how this helps us in the question of the statue, to give a taste of how it helps in broader mathematics. No prior knowledge of group theory, topology or category theory will be assumed.

**Angélica Osorno** Reed College

### Unifying Different Worlds in Mathematics

*3:00–3:35* PM

Category theory can be thought of as a language and a framework for making comparisons between different worlds. Often the comparisons consist of forgetting certain details in two worlds so that we see a sense in which the two worlds are the same deep down. In this talk we'll apply this principle to two constructions that might at first sight seem very different: the free group generated by a set, and the discrete topology on a set. We will find a property satisfied by both, that we can make precise using category theory. Using this example and many others, we will show that category theory allows us to unify concepts from different areas of mathematics and work with them as if they were the same. No prior knowledge of group theory, topology or category theory will be assumed.

**Emily Riehl** Johns Hopkins University

### From Arithmetic to Category Theory

*3:45–4:20* PM

You probably know that $a \times (b + c) = a \times b + a \times c$ because of the distributive law of multiplication over addition. But why is it true? Typically we might prove it using repeated addition, or perhaps by a geometric method involving areas of rectangles. In this talk we'll prove it via a roundabout method that takes us on a tour through several deep ideas from category theory including categorification, the Yoneda lemma, universal properties, and adjunctions. The point here isn't to re-prove a familiar result, but to show how ideas in category theory can stem from familiar basic math and yet be generalized to encompass wildly diverse examples. No prior knowledge of category theory will be assumed.

# Contributed Paper Sessions

## A Number is Never an Answer: Developing Mathematical Thinking and Communication Through Writing

### Part A: Thursday, August 2, 1:30–5:45 PM, Governor's Square 10, Plaza Building

**Organizers: William Gryc** Muhlenberg College
**Linda McGuire** Muhlenberg College

Many students only experience mathematics as a discipline of calculations. However students who take a quantitative job in an interdisciplinary field need to be able to clearly communicate mathematics to a lay audience via writing. In this session we invite instructors to discuss their use of writing assignments in their mathematics courses that develop these transferable skills.

**Paula R. Stickles** Millikin University

#### Using Writing Assignments in a Quantitative Reasoning Course

Many students have trouble communicating about mathematics and connecting mathematics to the outside world. Using writing assignments is a way to encourage them to make the connections. In this session we will look at a series of writing assignments used in a quantitative reasoning course. We will share students' reactions, assessment rubrics, lessons learned, and next steps.

**Sandra Fital-Akelbek** Weber State University
**Jean Norman** Weber State University

#### Writing With Numbers

Writing With Numbers is an entry level, cross-disciplinary course that combines mathematics with humanities. The course explores topics from mathematics that convey the beauty and utility of mathematics and illustrate its application to modern society. The course also provides language and develops communications skills to speak accurately about mathematical concepts in a way a layperson would understand and practice in writing about these concepts. The topics from mathematics fulfill the quantitative literacy requirement that prepares students for real-life problems.

**Emilie Hancock** Central Washington University
**Gulden Karakok** University of Northern Colorado

#### Developing Metacognition Through Process-focused Writing in an Inquiry-Based Learning Classroom

As students learn to problem solve, they must also develop metacognitive tools to manage and regulate their problem-solving process. To foster "process-focused" metacognition utilized by mathematical thinkers and problem solvers, Inquiry-Based Learning classroom practices and an adapted version of "portfolio" problems were implemented in a content course for pre-service elementary teachers. On multiple occasions during class, students worked together on a problem typically more open-ended than usual course work. Students then submitted individual portfolio write-ups, providing their scratch work, a revised solution approach that included mathematical justification and reasoning, as well as their judgement and decision-making processes during the entire problem-solving attempt. In this talk, we describe how a process-focused (instead of product-focused) classroom culture and explicit reflection practices mediated by in-class portfolio problem-solving sessions and write-ups supported students' process-focused metacognition while problem solving. The problems, portfolio structure, and student interview reflections will be shared.

**Hope Essien** Malcolm X College

#### Incorporating Written Communications into Mathematics Deepens Students Learning Outcome in General Education Mathematics.

Encouraging an active learning environment for student presents challenges, particularly in a mathematics classroom. Recent curriculum redesign recommends cognitive mathematical learning tasks for all students to utilize multiple modes of communication to deepen students learning outcome. The aim of this project is to develop and create a mathematics learning space that facilitates active learning using activities to foster connections that enhanced student learning through writing in mathematics. According to NCTCM (2000), "communication is an essential part of mathematics and mathematics education. It is a way of sharing ideas and clarifying understanding. Through communication, ideas become objects of reflection, refinement, discussion, and amendment." Writing in a mathematics classroom is vital in the teaching of mathematics since it assist the instructor and student to assess whether students understand mathematical concepts presented. According to Quealy (2010), writing in mathematics is important since it allows students to become active learners and encourages deeper understanding of mathematical concepts. Additionally, Pugalee (2004) and Wilcox et al., (2011), contends that writing can be an effective tool in mathematics education and writing may have a positive effect on students' ability to understand mathematics. This investigation demonstrates that there is a relationship between writing in mathematics (active learning) techniques and students developing mathematics skills and deepening students

learning outcome. It affirms that using written communication in mathematics utilizes students centered learning style and encourages students to reflect on materials covered.

**Nell Rayburn** Austin Peay State University

### Using Writing Ideas from English Teachers in a History of Mathematics Course

The pedagogical resources created by our colleagues in language and literature departments are an often overlooked but very valuable source of writing prompts, assignment guidelines, and grading strategies. The talk will focus on the author's experience with a critical book review assignment and a primary source analysis assignment in a history of mathematics course for undergraduate seniors and first year graduate students. We will also look at guidelines for writing mathematical definitions inspired by our language arts colleagues.

**Abigail C. Bishop** Iona College
**Benjamin Gaines** Iona College

### Writing Short Essays in a Core Mathematics Course

We will discuss the use of calculus in conjunction with real-world data and with an eye on social justice as the basis for writing a short essay for a mathematics course that is part of the core curriculum at Iona College. Two assignments used in Business Calculus and Calculus I courses will be presented: one based on a measure of income inequality, the Gini coefficient, in a particular county and the other on educational attainment in rural and urban counties in a state. We will talk about the creation, implementation, and both the successes and the challenges of including these writing assignments as part of a core mathematics course.

**Michelle Ghrist** Gonzaga University

### Exploring Integral Calculus Through Applied Writing Assignments

In a recent integral calculus course, I assigned a series of biweekly writing assignments. These assignments ranged from having students look for and discuss transcendental functions in their everyday lives, to exploring the Great Molasses Flood of 1919, to applying infinite series to understand IRS tax code. My main intentions were to expose students to some deeper applications of the mathematics that we were studying and to help them develop better technical communication skills. I discovered that many students had never done any such writing; early on, I often found myself questioning whether I had made a mistake in assigning this work, as the start-up cost for some students as well as my time investment was rather large. However, I saw dramatic improvement in students' writing throughout the semester, and many students commented that they really enjoyed seeing the applications and recognized the utility of the skills that they had learned and developed. In this talk, I will share the assignments and discuss some of my observations, to include some of my triumphs and struggles.

**Joy Becker** Wartburg College

### Writing through Applications in Multivariable Calculus

Multivariable calculus includes applications across various disciplines. In order to help students make connections to their majors and/or career choices, they complete an application project, which includes a significant amount of writing. Part of the project involves writing a multivariable calculus problem, solving it, and explaining the solution. In addition, students are asked to write about the particular application they chose, making connections between the course and their major and/or career interest. Students also write a reflection on the course and those connections, developing their mathematical and general communication skills. This talk will explain the project and goals in more detail and provide examples of student work.

**Malgorzata A. Marciniak** CUNY LaGuardia Community College

### Writing Intensive Upper Level Math Courses for Engineers and Computer Scientists

Title: Writing intensive upper level math courses for engineers and computer scientists Abstract: Motivated by the fact that engineers and computer scientists spend about half of their time either reading or writing, I convert Calculus 3 class into a writing intensive course at the large, urban community college. Participation in a college seminar "Writing in the Disciplines" organized by the Center of Teaching and Learning helped in preparation of class materials such as potential topics and grading rubrics. Suggestions and examples provided by faculty from humanities encouraged feedback loops and resubmissions. Following the seminar guidelines, I prepared informal and formal assessment questions that were assigned at the beginning of the semester and then assigned a longer writing assignment for students. Students wrote several short paragraphs (about 250 words) and one longer (about 600 words). Using written assignments was very helpful in understanding students' level of comprehension. Often the writing exposed what misunderstandings and misconceptions students are carrying. Those misconceptions often remain hidden when only equations are shown for a solution of a problem. In particular, non-native speakers benefit from written assignments the most, however, their initial resistance may be discouraging. Those students often know how to solve math problems but have difficulties finding phrases to explain their work. Since spoken language is the most common for basic communication at academia and working place, this issue needs to be addressed as soon as possible while they are still in college.

**Maria Neophytou-Foster** Belmont University

**Assessing Department SLOs in a Linear Algebra class**

In this presentation I will discuss how I incorporated the assessment of some of our department Student Learning Outcomes (SLOs) in my Linear Algebra class, and how the students benefited from this. The relevant outcomes involve the ability to model and analyze real world problems, as well as the ability to communicate mathematics orally and in writing. Covering all the fundamental Linear Algebra material in a one-semester class is definitely challenging, but helping the students see how to apply and communicate their knowledge and practice these important skills can be done without using up a lot of valuable class time. It can also be a very meaningful and enjoyable experience for the students!

**Karin R. Saoub** Roanoke College

**Critical Thinking and Writing Development through Project and Paper Scaffolding in a Liberal Arts Math Course**

Graph theory can be taught from many different viewpoints. Some courses emphasize the theoretical aspects (such as connectivity, planarity, or isomorphisms) whereas others may focus on modeling real-world problems (such as delivery routes, shortest paths, and matchings) or solution algorithms (such as Dijkstra's Algorithm for shortest paths, Nearest Neighbor for Hamiltonian cycles, or Kruskal's Algorithm for minimum spanning trees). How do you teach all three aspects of graph theory in a general education course where students have little to no background in mathematical proof? In this talk, we will discuss assignments that progress from the basics of modeling a problem using graph theory to producing a polished final paper. The assignments guide students in their development of writing about mathematics, from describing the solution techniques applied to critiquing the solution in terms of the real world problem being solved.

**Cory Johnson** California State University, San Bernardino

**Write, Revise, Repeat: Improving Student Writing**

Many universities have a writing requirement for undergraduate students, but students may not explore writing styles specific to their own discipline. At some point, a mathematics student will be expected to make the transition from writing computational based solutions to writing formal proofs. This transition in writing style often occurs simultaneously with learning new content. In this talk, I will discuss how I scaffolded assignments with the goal of improving my students' writing. Assignments emphasized writing as process by requiring multiple revisions which were then collected for a portfolio that highlighted a student's growth. Although these assignments were implemented in an Abstract Algebra sequence, the ideas can be applied to any mathematics course. I will also share some ideas for assessing student writing.

**Jeff Johannes** SUNY Geneseo

**Writing Across the Mathematics Curriculum.**

Although every course I teach is different, writing plays a prominent role in many of them. This ranges from writing about philosophy of mathematics in a liberal arts course to explaining the central topics in courses for elementary teachers. I will also discuss examples for majors in introductory seminars along with capstone courses and a course that borders on graduate work. While I do not use writing in all of my courses, in every instance in which I do it promotes reflection and personal responsibility for the course ideas.

## Part B: Friday, August 3, 9:00 AM–12:15 PM, Governor's Square 10, Plaza Building

**Lauren DeDieu** University of Calgary

**Reflective and Expository Mathematical Writing Assignments**

Incorporating mathematical writing into the curriculum can be an incredibly value tool. In addition to helping students better understand course content, precise mathematical writing can train students to become logical thinkers and construct rigorous arguments. Mathematical writing can also be used to teach students how to become better communicators and better learners. In this talk, I will describe the reflective and expository writing assignments I have used in my undergraduate mathematics courses. In expository writing assignments, students are asked to explain complex mathematical ideas to non-experts. The audience is usually specified ahead of time (e.g. a classmate, boss, community member). In reflective writing, students reflect on their personal thoughts, feelings, and experiences that relate to the content being learned. Reflective writing can help students stay accountable for their own learning, boost motivation, and create a channel of communication between the instructor and students. In my talk, I will share my motivation for using these assignments, their potential benefits, as well as students' reactions to these types of assignments.

**Zoe Dai** Alma College

**"Math $\neq$ Writing ?"**

An astonishing number of college students who take math courses have this preconceived notion of "math $\neq$ writing". Is this true? Where did this notion come from? And how is it affecting the teaching and learning of mathematics? In this talk, we will explore answers to these questions. Good mathematical writing is a habit that can only be formed over time, from an early stage, through very intentional training. The author will discuss different strategies employed in courses at various levels that have demonstrated effectiveness.

**Livvia L. Bechtold** University of Colorado Denver

### Journaling in Trigonometry

In this talk I will illustrate how I use writing in my Trigonometry class to help students develop a mathematical language, improve problem solving skills, and empower them to be independent learners of mathematics. I will highlight how I use journal entries, reflections, group writing, and discussions held in class or on a computerized learning system to achieve these goals.

**Kelleen Bonomo** Grove City College

### Using Writing to Aid Pre-Service Teachers' Understanding and Explanations of Mathematical Concepts

Many Elementary Education majors enter college with the view that mathematics is purely procedural. Some of them lack any knowledge of the conceptual side of math, and are unable to explain such things as why the algorithms for the basic operations on whole numbers work the way they do, or why common denominators are necessary for adding fractions but not for multiplying, or why probabilities can't be negative. In my mathematics content courses for elementary PSTs, I use short writing assignments to build these students' understanding of mathematical concepts and to help them become better at communicating mathematical explanations. This talk will focus on what I look for in grading these papers, as well as the improvements that I see (or don't see) over the course of a year with these students.

**Kristin Kuter** Saint Mary's College
**Charles Peltier** Saint Mary's College
**Ranjan Rohatgi** Saint Mary's College

### Case Studies in Statistics for Business Students

At Saint Mary's College, business students are required to take two semesters of statistics. Throughout the second semester, students work in groups to write four managerial reports on different applications of statistics to business scenarios. A main goal of these reports is to teach the students how to communicate their results to an audience not well-versed in statistics. In this talk, we will discuss the ways in which we embed these case studies in our course.

**Lanee Young** Fort Hays State University

### Writing in Statistics Class

Many times in our introductory statistics class, students become quite proficient at using technology to calculate p-values when performing hypothesis tests. Having the ability to calculate an answer is not the same as understanding what the answer means. Through a semester-long project students work in groups to collect, organize, and analyze data. Students are required to write a project proposal, methods, and present results in written format. Through a modified poster session, students present their results both in written and spoken form through discussions with their peers. This process gives many students a taste of research and presenting results that they may not have had up to this point.

**Magdalena Luca** Mass College of Phamacy & Health Sciences

### Math Without Writing: Like a Car Without Wheels

The importance of writing in the mathematics classroom cannot be overstated. Throughout the writing process, students clarify their understanding of mathematics and improve their communication skills. In addition, it is vital for all mathematicians to be able to write clearly and effectively since they have to communicate their findings to a variety of audiences. Proficiency in writing can only be accomplished through active practice. Unfortunately, students are rarely required to write in introductory or interdisciplinary math courses, such as those offered at our university. In this presentation I will discuss the use of writing assignments in Business Math, Calculus, Statistics and Biostatistics, and I will also present specific writing examples that range from short answers used in quizzes to in-depth writing analyses used in projects.

**Alexander Halperin** Salsibury University
**Colton Magnant** Georgia Southern University

### Two Introductory Mathematical Writing Assignments

Writing assignments in any mathematics course always present several challenges, particularly in lower-level classes where the students are not expecting to write more than a few words at a time. Developed based on strategies from various sources, the two small writing assignments included in this presentation represent a gentle introduction to the writing of mathematics and can be utilized in a variety of low-to-middle level mathematics courses for majors and non-majors. The first prompt requires students to solve a problem analogous to the Seven Bridges of Konigsberg; the second asks students to use combinatorial proofs to calculate the probability of five different poker hands. Our goals for each assignment were to familiarize students with the mathematical writing process and emphasize the importance of logical, clearly stated, succinct arguments. We found that student enthusiasm improved on the second assignment, leading us to believe that prompts of this nature may be a suitable way to introduce students to mathematical writing.

**Haley A. Yaple** Carthage College

**"Dear Calculus Consultant:" Projects where Students act as Experts**

In their future mathematical careers, our students will not only have to do correct work, but also convince others that their work is both correct and meaningful. Given an appropriate framing, writing projects can require students to practice clear and convincing explanations of mathematical results and methods. In my calculus classes, projects are assigned as requests for help from entrepreneurs and inventors, who need advice of a mathematical nature. In this talk I will give several examples of project prompts from first- through third-semester Calculus and highlight learning objectives and assessment methods for these assignments.

**Andrew George** Penn State Erie

**Writing on Calculus Exams: Forcing Students to Reason and Communicate**

The College Board's recently designed Mathematical Practices for AP Calculus (MPACs)—upon which the AP Calculus curriculum framework is built—include reasoning with definitions and theorems (MPAC 1) and communication of mathematics (MPAC 6) (*AP Calculus AB and AP Calculus BC Course and Exam Description Effective Fall 2016, The College Board, 2016).* Influenced by my involvement with the AP Calculus program over the past 16 years, I have long been a proponent of emphasizing written communication and reasoning in lower-level college mathematics courses. In this talk, I will illustrate ways I have used writing on Calculus exams as a means of gauging and sharpening students' understanding, as well as a way of "forcing" students to reason and communicate beyond their computations.

# Advancing Women in Mathematics: On the Ground Initiatives

## Thursday, August 2, 1:30–5:25 PM, Governor's Square 14, Plaza Building

**Organizers: Della Dumbaugh** University of Richmond
**Heather Russell** University of Richmond

This session focuses on how programs advancing women in mathematics take shape on the ground. Speakers will discuss critical project components including aims, intended audience, implementation, replication, and scaling. This session provides a broad array of ideas that together form a frame for how to begin—or continue—a dedicated effort to move women forward in mathematics.

**Lauren Keough** Grand Valley State University
**Feryal Alayont** Grand Valley State University

**Being Intentional: Increasing Success of Women in the Mathematics Program at GVSU**

The Mathematics Department at Grand Valley State University has strived to mentor math majors for career and graduate school opportunities for a long time. However, there was no structured mentoring program until recently. The Increasing Success of Women in Mathematics program targeted beginning undergraduate women interested in math. Our goal was to set up a layered approach to mentoring which tapped into current senior women math majors as mentors and alumna and faculty members as role models. A wide range of events were organized to give students information about what to do during their undergraduate studies, and help them plan for careers and graduate school. We will describe what we learned: successes and failures, and report on student feedback about the program.

**Felicia Tabing** University of Southern California
**Cindy Blois** University of Southern California

**The WoMentoring Group**

Mentorship is a key pillar of support for undergraduate and graduate students in mathematics as they navigate the academic world, especially as they are making key decisions about their future. In addition to pairing graduate and undergraduate students as mentors and mentees, we facilitate monthly group meetings for all participants of The WoMentoring Group, alternating with one-on-one mentor-mentee meetings. Each group meeting has its own theme (e.g. "The Career Workshop" ) and mentors and mentees are given small "homework" tasks to complete during their pair meetings. The goal is to provide more support and structure for the mentor-mentee relationships, in addition to facilitating a close-knit community of women in math.

**Rachelle DeCoste** Wheaton College (MA)

### The Career Mentoring Workshop (CaMeW)

CaMeW was founded in 2007 to support women who are finishing their PhDs in the mathematical sciences. Most graduate students are receiving PhDs from R1 institutions and working with advisors who have spent their entire career at R1 institutions. However, most PhDs in mathematics do not go on to careers at such institutions. We developed our workshop to address the needs of women who were looking to establish successful careers in academia, at all kinds of institutions. The 3-day workshop is comprised of interactive sessions through which participants learn about career options, are encouraged to determine their own ideal postdoctoral career path and form supportive relationships with peers and faculty mentors on whom they can call in the future. We will discuss successes and challenges of running CaMeW, including funding challenges, the benefits of community-building and mentorship, and the need to support women in mathematics at all levels of their careers. We will also briefly mention other initiatives aimed at increasing numbers of women in STEM, more broadly.

**Alessandra Pantano** UCI
**Natalia Komarova** UCI
**Patrick Guidotti** UCI

### Building a Community of Peers

With the generous support of the WATCH US grant *"Building a Community of Peers"* , the Mathematics Department at UC Irvine has implemented a variety of initiatives aimed at strengthening the network of women PhDs, by developing connections among fellow students and faculty, and providing positive role models from both industry and academia. This multifaceted program aims at simultaneously addressing multiple barriers to the success of women in mathematics. Activities range from paint nights to establish and cultivate new friendships among women in STEM, to career panels to support women's professional aspirations in the mathematical field. In this talk, we will discuss the outcomes of these activities and distill a number of lessons learned, in the hope to contribute to the funds of knowledge of best practices for supporting women in STEM in college campuses across the country.

**Della Dumbaugh** University of Richmond
**Heather M. Russell** University of Richmond

### Leveling Up: Building Community and Confidence

Malcolm Gladwell's bestseller *David and Goliath*: *Underdogs, Misfits and the Art of Battling Giants* provides surprising insight into successful strategies for women in mathematics. In particular, Gladwell points out that over half of all American students who begin in STEM fields drop out after their first or second year. Gladwell's work suggests that women may feel deterred from continuing their undergraduate studies in mathematics because they feel unconfident and uncomfortable in the math classroom. Inspired and supported by a NSF "Watch Us" INCLUDES grant, we aimed to redress this issue by intentionally focusing on the Multivariate Calculus and Linear Algebra classrooms. We offered dedicated Multivariate Calculus and Linear Algebra study groups in the context of broader initiatives designed to create positive peer relationships and build confidence from the inside out for our students at the University of Richmond. This talk discusses the details of these initiatives and the outcomes.

**Rebecca Segal** Virginia Commonwealth University

### INCLUDES WATCH-US Mini-grant: C3PO (Core knowledge, Community, and Confidence through a Programming Overview)

At Virginia Commonwealth University, we offer a PhD in Systems Modeling and Analysis which is an interdisciplinary program housed between the Department of Mathematics and Applied Mathematics and the Department of Statistical Science and Operations Research. This program was created 10 years ago to address the need for a degree which focuses on mathematical science approaches to modeling solutions for complex applied problems. Our students arrive at this program from a variety of backgrounds and take jobs in both industry and academia following graduation. We currently have about 30% women in the program and their performance and graduation rates match that of the men. We would like to increase the percentage of underrepresented groups in the program, including women. We hope that creating a supportive culture along with content resources will make the program desirable for all groups. In an effort to put students on equal foundation and provide them with tools to success, we are developing a series of programming tutorial courses to be delivered by current graduate students. These courses will be held during the summer, immediately before the start of the semester. We will report on the planned content, mentoring strategies, and goals for the project.

**Katrina Morgan** University of North Carolina at Chapel Hill
**Francesca Bernardi** University of North Carolina at Chapel Hill

#### Mentoring, Outreach, and Professional Development: Activities of the AWM Student Chapter at UNC-Chapel Hill

The Association for Women in Mathematics Student Chapter at the University of North Carolina at Chapel Hill has developed several programs, activities, and events over the past 3 years designed to advance women in Mathematics. The chapter organizes a Mentor Network connecting undergraduate mentees with graduate student mentors, runs the free summer day camp Girls Talk Math, holds Meet the Speaker teas and lunches with visiting female speakers, invited a female mathematician to give a public lecture at UNC, and held the first AWM Triangle conference last spring (funded by an NSF INCLUDES: WATCH US mini grant). This talk will discuss the goals of these programs, assess their efficacy, and share ideas on replicating these programs at other institutions.

**Kaitlyn Phillipson** St. Edward's University
**Jason Callahan** St. Edward's University
**Carol Gee** St. Edward's University

#### Women Empowered through Graduate Opportunities Awareness Transformation (weGOAT)

The goal of the weGOAT project is to increase our majors' awareness of, preparation for, and subsequent success in doctoral programs in the mathematical sciences. Building on our existing four-semester research sequence, we are adding outside speakers from graduate programs and non-academic careers, social activities, workshops on preparing for graduate school, and trips to conferences and graduate programs. We have also introduced a new outreach initiative by designating a First-Year Math Major Mentor in order to start building the support network early in students' careers.

**Catherine Berrouet** Florida Atlantic University
**Anae Myers** Florida Atlantic University
**Angela Robinson** Florida Atlantic University
**Jessica Thune** Florida Atlantic University
**Yuan Wang** Florida Atlantic University

#### Dare to BEE

If you are looking for honey, you will likely find bees. Likewise, as budding women mathematicians drawn to the sweet thrill of proving theorems, we often feel the sting of isolation and uncertainty along the way. "Dare to BEE" (Be Exceptionally Encouraged) is a small scale mentoring program where graduate students (Big Bees) mentor undergraduates (Little Bees) through events that emphasize community and development such as: visits by early career and established female mathematicians, panel discussions, recruiting students at the AMS Annual Meeting, hosting an AAUW Salary Negotiation Workshop, sponsoring joint poster sessions and lectures with AMS and SIAM, and hosting socials. We discuss our approach to success and, as busy BEEs do, consider the future outlook for expanding our hive. Oops! We mean the scope of our program.

**Alice Nadeau** University of Minnesota
**Kim Logan** University of Minnesota
**Harini Chandramouli** University of Minnesota

#### Mathematics Project at Minnesota

The Mathematics Project at Minnesota (MPM) is a four-day workshop for undergraduate women at the University of Minnesota preceding the beginning of spring semester. MPM is designed to address the lack of female representation in the math major and in math classes (both in the student body and within the instructor pool) which results in female students feeling isolated and disengaged with the mathematics community at the University. The workshop is facilitated by women graduate students, post-docs, faculty, and junior/senior female undergraduate math majors who serve as mentors and role models both during the workshop and as the participants progress into the major. The four main themes of the workshop are: (1) community building, (2) diversity in mathematics, (3) professional development, and (4) mathematics in practice. MPM was adapted from a program at Grinnell College (Iowa) for underrepresented students in the sciences, and the developers believe the MPM model is transferable to other institutions.

**Alison Marr** Southwestern University

#### Hidden No More Lecture Series

As part of a mini-grant from the NSF INCLUDES "WATCH-US" team, Southwestern University hosted a three-part lecture series in Spring 2018 titled "Hidden No More: Stories of Triumph, Excellence, and Achievement in Math and Computer Science." Each speaker was a female from an underrepresented group with a Ph.D. in mathematics. The talks included both the speaker's story to becoming a mathematician as well as what kind of math she does now. In this talk, I will share some of the logistics involved in making the series happen, some reflections from students that participated, and some lessons learned along the way.

**Jessica Beck** University of Tennessee
**Natalie Lemanski** University of Tennessee
**Nina Fefferman** University of Tennessee

#### Developing Peer Networks by Producing Videos that Highlight the Careers of Women in Math

Barriers such as lack of confidence and isolation limit the number of women pursuing careers in math. In order to break down these barriers, we will host a networking workshop from May $16^{th}$ to May $18^{th}$ in which early-career participants (students) collaborate to conduct and film short interviews of mid- and late-career role models (mentors). Fifty applications from interested students were received, and seven participants were chosen. Three mentors were selected from visiting faculty. After the event's conclusion, we will upload the videos to the Women Do Math Facebook page and Youtube channel. We will also request feedback from participants which we will be able to present in August. A year after the event, we will ask participants about their career progress and whether they've maintained connections with those they met at the event. The interview structure creates a framework through which participants can learn from mentors and expand their peer community, providing an essential support system. The proposed program is clearly scalable to include a much larger number of participants, provided the ratio of mentors to students can be maintained.

## Best Practices and Innovation in the Teaching of Discrete Mathematics

### Friday, August 3, 1:30–4:45 PM, Governor's Square 10, Plaza Building

**Organizers: Zsuzsanna Szaniszlo** Valparaiso University
**Ágnes Bércesné Novák** Catholic University, Hungary
**Peter Pazmany** Catholic University, Hungary

This session seeks presentations about innovative approaches to the teaching of Discrete Mathematics, the course generally required for computer science majors. Presentations could include illuminating projects and exercises, new approaches to the traditional curriculum, and ways to address new, interdisciplinary student populations. Presentations should focus on easily adaptable models and should discuss how stated learning objectives are attained.

**Kathleen Shannon** Salisbury University

#### Implementing WebAssign in Discrete Mathematics

This paper will report on the progress of incorporating WebAssign in a freshman/sophomore-level Discrete Mathematics course serving mathematics and computer science majors, as well as mathematics minors from a number of disciplines, at a comprehensive university. The speaker is the author of *Discrete Mathematics: A Brief Introduction*, which has been used as the primary text for Math 210: Discrete Mathematics at Salisbury University since Fall 2003. Over the 2016-17 academic year, the author worked with WebAssign to create an e-version of the text and to program homework exercises for WebAssign's automatic grading system. She is currently engaged in preparing "Watch it" videos to accompany the text and problems. This paper is an update of the paper given at JMM. In it the author will discuss the process of converting the text, using WebAssign for Discrete Mathematics, and the general nature and content of the course.

**Breanne Garrett** William Penn University
**Elizabeth Overturf** William Penn University
**Kiera MacPherson** William Penn University

#### IBL in Discrete Mathematics

IBL in Discrete Mathematics At William Penn University (WPU) a variety of programs require a Discrete Mathematics course. Because of this, our course is somewhat unique to our school and the learning outcomes differ slightly from what most textbooks propose. Number systems and introduction to matrices are two such topics that deviate from the traditional path. A deferment of formal proof-writing until a subsequent course is another. With some Inquiry-Based Learning (IBL) experience, we chose to dive in and write an IBL workbook for our Discrete Mathematics course. Our school has adapted a STREAM model: STEM + Relationship building (the R) + Artistic creativity (the A). This new model seemed to fit both the goals of the course and of the broader University. The workbook has an opening task for each section that poses a real-world question to ponder. Then a short reading follows, including interleaved definitions, properties, examples and exercises. Each section concludes with a prompt to revisit the opening task to attempt a solution using information from the reading. This talk will highlight the challenges we faced writing the material and properly conveying its message to the first group of students, the outcomes of this first semester (both qualitative and quantitative), and how we revised the course in order to move forward this fall. We would be delighted if instructors would like to borrow and test all, or part of, the workbook for their courses.

**Kurt Ludwick** Salisbury University

**Count That Tune: Teaching Counting With Musical Examples**

Counting is a standard topic in discrete mathematics courses. Various types of counting problems can be motivated through musical considerations. I have successfully used musical examples to help students learn a variety of counting concepts and techniques. In this talk, I will demonstrate music-based examples of combinations, permutations, multisets, and recurrence relations.

**Zsuzsanna Szaniszlo** Valparaiso University

**Discovering Binomial Coefficients**

Permutations and Combinations are often taught as separate topics. We present a unified development of these topics. Students usually learn permutations without repetition from the multiplication principle. We present an activity that can be used to guide students in the discovery of the formulas for permutations with repeated elements. Combinations are presented as a special case of repeated permutations. Students have good success remembering the process in later classes and tests.

**Feryal Alayont** Grand Valley State University

**Short Case Studies to Improve Student Understanding of Intricacies of Counting Problems**

Counting problems in Discrete Mathematics provide great examples of real-life applications as well as multiple solution methods and difficult to identify issues in solutions. This presentation will focus on the use of short case studies based on analyzing previous student solutions as an instruction method to improve student understanding of counting methods and flexibility in using these methods. Examples of case studies and student responses will be shared.

**Teena Carroll** Emory & Henry College

**Using the Boards of Board Games to Motivate Graphs**

A potential pitfall in the first days of graph theory is drowning students with a sea of definitions without sufficient motivation. Many board games use graphs (or maps which can be converted to planar graphs) as an integral part of the gameplay. I will discuss the various roles that board games play in my discrete mathematics class– a way of generating interest, creating exposure to the breadth of applications of graphs, sources of counting problems, and a way of creating research questions.

**Anthony Bosman** Andrews University

**Video Project for a Discrete Math Course**

Our discrete math class serves to introduce students studying mathematics, computer science, and data science to a number of important mathematical structures and proof techniques. In this talk I share an end of the semester research project that had the students study a topic in small groups, write up a report on it, and create a short YouTube video to communicate what they learned to a popular audience. The project was designed to let the students further explore a topic of interest to their field and help the students develop competence and confidence in communicating mathematics. We'll discuss how the project met these goals, challenges the project presented, and how I plan to modify the project for future versions of the course.

**Donna Beers** Simmons College

**Uncovering critical nodes in a supply chain: Connecting graph and network theory to supply chain risk management**

In this talk we describe a new module for enriching the graph theory portion of our discrete mathematics course. Our module derives from our experience, this year, in using the Bloomberg Terminal while on sabbatical. Its focus is to show how graph and network theory may be applied to uncover critical nodes within a supply chain. 'Assurance of suppliers' is a fundamental tenet of supply chains. Students appreciate the importance of ensuring the safety of our food and drug supply chains. More generally, different suppliers potentially pose different risks to a purchasing company. For example, a single-source supplier poses a huge risk in the case of a natural disaster, when the supplier's operations may be shut down, while a supplier whose compliance systems are inadequate pose another kind of risk. We will define supply chain risk and describe the specific graph topics covered in the module (e.g., betweenness centrality, typology of ties between companies, flows between companies). Because undergraduates at many schools now have access to the Bloomberg Terminal, we will discuss how students can use this tool to construct a particular company's supply chain to three tiers and beyond.

**Jordan Tirrell** Mount Holyoke College

**Writing and Revising to Conquer Proofs in Discrete Mathematics**

One of the biggest challenges of teaching Discrete Mathematics is introducing proofs to students without pre-existing habits for logical reasoning. In this talk, we will discuss an approach where students are asked to write a detailed solution for one challenging problem each week. Some of these problems are difficult even for veterans of mathematics competitions, and they focus every student's attention on logical structure and writing clarity. With ample time, hints, and revisions, these problems are accessible to under-prepared students and allow them to make surprising developments as proof writers.

**Gregory V. Bard** University of Wisconsin—Stout

### Counting the Keyspace of WWII's Enigma, and What it Can Teach us About Modern Cryptography

Most of us are aware of the Engima cipher machine, whose several variants were used by the Germans in World War II. Computing the size of the keyspace is a moderately difficult combinatorics problem, suitable for a discrete mathematics course. Because the cipher works on an alphabet of size 26, and not binary, and especially because of the Enigma's plugboard, the computation is non-trivial. An often overlooked fact is that consecutive letters were never connected on the plugboard. For example, S could be connected to any letter except R, S, and T, or could be left unconnected. While this appears to be not widely known, it does impact the size of the keyspace. The British cryptanalysts even added a circuit to the *Bombe* cryptanalysis machines, called Consecutive Stecker Knock Out (CSKO), to take advantage of this fact. During this talk, I will show the (single-slide) solution to the computation of the size of the keyspace of the two most common Enigma variants, without taking into account the above fact. Then I will explain how the shrinkage in the keyspace as explained above can be quantified, which is somewhat more difficult, but not very difficult. Moreover, I will briefly describe how these calculations can be used to explain to students in computer science that the cryptanalysis done in World War II by the Poles, and later the allies at Bletchley Park, was not what cryptologists call *brute-force cryptanalysis*. It was not the case that every possible key was checked in a loop. This brief realization demonstrates the modern principle that a large keyspace is *necessary* but not *sufficient* for the security of cryptographic systems—a fact that remains true in the modern era.

## Encouraging Effective Teaching Innovation

### Part A: Thursday, August 2, 9:00–11:55 AM, Governor's Square 12, Plaza Building

**Organizers: Susan Crook** Loras College
**David Failing** Lewis University
**Russ Goodman** Central College
**Mami Wentworth** Wentworth Institute of Technology

This session will consist of presentations of demonstrably effective and innovative classroom techniques that address the reasoning behind, design, and implementation of resources or activities. This may include whole course techniques (not necessarily original to the presenter) or drop-in activities to bolster student learning and reflection in any course. Materials will be shared after the session at: http://mathfest2018.davidfailing.com.

**Keith Carlson** University of Central Florida

### Exploration of Methods in the Teaching of Pre-Calculus

There is a direct correlation of student competence in pre-calculus algebra and their success in calculus. Failure in calculus essentially prevents the completion of studies in a wide variety of subjects. At the University of Central Florida we have been concerned with the preparation of students for calculus. We have explored teaching that emphasizes those topics that frequently create difficulties for students learning calculus. In this talk we will outline the details of our work.

**Gowribalan Ananda Vamadeva** University of Cincinnati

### Spicing up a Developmental/First Year Algebra Classroom

Are you afraid of being the lone passenger on the reform train? This session introduces effective methods used to supplement a traditional curriculum with conceptual ideas in a Developmental/First year algebra setting. Faculty will be given strategies to have an enlightening classroom environment that balances the needful skills with much needed theory. Many departments across the country are still using traditional textbooks and curriculum in their algebra classrooms. Every day, Faculty find a hard time having their voices heard in a departmental setting to instill curriculum that promotes comprehending concepts in their courses. Attending this session will enable interested and deprived faculty create a learning environment that uses concepts as a central theme within the framework of a traditional curriculum. Faculty will benefit from acquiring the balance between teaching essential skills using a variety of perspectives and communicating the importance of the concepts that lie within

**Courtney Fox** Clermont Northeastern Schools

### The Integration of Mathematics and Science: A Plan for a High School Integrated Pre-Calculus and Physics Course

This paper explores the integration of mathematics and science as a means to improve learning for high school students. Scholars have acknowledged the benefits of integration for over 50 years, but we have failed in large measure to adopt an integrative curriculum in public high schools across the country. This work provides a corrective to this problem by creating a practical curriculum for an integrated Pre-Calculus and Physics course with suggestions for implementation in any school.

**Ciarán Mac an Bhaird** Maynooth University

### Effective Methods forImprovingStudent Retention and Progression

Worldwide, increasing numbers of students are attending Higher Education (HE) without the mathematical skills that they require to succeed in their chosen courses. As a consequence, these mathematical issues are often identified as one of the main factors in high student drop-out rates across a wide range of courses which have some level of mathematical or statistical content. Saxe and Braddy (2015), in a report for the MAA, note that '... the high rate of failure in post-secondary mathematics classes is an embarrassment to our profession. It is a major contributor to increased attrition rates, and it lengthens time to degree at all types of post-secondary institutions. Mathematics courses are the most significant barrier to degree completion in both STEM and non-STEM fields.' Internationally Math Centers have been established to try and tackle student math deficiencies in the transition to, and first years of HE. I will give an overview of how Math Centers operate in Ireland and the UK, with the main focus on drop-in activities and the range of other free supports and resources we provide for students to assist with their study programmes. The effectiveness of these initiatives is often related to the systems of continuous assessment and small group teaching in place. I will present research which indicates that appropriate student engagement with Math Centers can improve student retention, progression (especially for the 'weakest' students) and the student learning experience. I will close by describing our drop-in initiative for secondary schools and how trainee teachers have this experience incorporated into part of their course.

**Erin R. Moss** Millersville University of Pennsylvania

### Supporting College Algebra Students' Study of Mixture and Motion Problems

Few topics are met with more dread in college algebra courses than word problems. Scenarios involving mixing solutions and uniform motion are often included in the college algebra curriculum. Once these problems are understood, they are not particularly difficult—they share a similar multiplicative structure and a need for the student to interpret results of solving for an unknown variable in the context of each problem. Yet students can become paralyzed with anxiety when faced with one of these problems and rely on desperate, nonsensical means of combining variables and quantities rather than using prior knowledge about these situations and thinking clearly. Before attempting each of these problem types, I have students answer some conceptual questions in small groups about various mixture and motion scenarios. They use what they already understand to draw conclusions about how the quantities in a problem are related or to determine reasonable estimates of a final answer. Each of these questions can be answered without any knowledge of formulas—just common sense and a little life experience. They provide significant insight into how we will proceed to solve the problems with more formality later. Once students realize they bring significant knowledge to the table and have become comfortable understanding all the "moving parts" of each type of problem, we can tackle their solution with fewer starts, stops, and panic-stricken faces.

**Jordan R. Hall** University of Colorado Denver

### Promoting the Use of Multiple Representations in the College Algebra Classroom

In this talk, we explore activities that allow for the use of multiple representations of mathematics during in-class active learning sessions and as components of assignments that students interact with outside of class. For example, we consider the case in which the learning goal is mastery of rational functions, a common sticking point for students. We propose a learning trajectory which relies heavily on multiple representations through the use of TACTivities (tactile activities for cooperative learning) and TECHtivities (dynamic, interactive activities which utilize Desmos). We discuss evidence gathered in our own classroom and best practices for implementation.

**Angie Hodge** Northern Arizona University

### TACTivities for elementary teachers

In this session, I will actively engage the audience in exploring tactile learning activities for elementary teachers. These activities can be used in methods courses for future teachers, content courses for future teachers, or in actual elementary classrooms. Shape classification and dimensional analysis will be the topics of focus with ideas provided for creating more TACTivities for your own classroom.

**Anna Seitz** Iowa State University
**Heather Bolles** Iowa State University
**Amanda Baker** Iowa State University

### Team-Based Learning Calculus

Team-Based Learning (TBL) is a specific form of active learning designed to collaboratively engage students in significant problem-solving tasks. By means of a flipped classroom, students are able to spend class time working in heterogeneous groups, applying fundamental concepts to a rich applied context. The TBL process provides peer scaffolding for students, extending the zone of proximal development as the designed tasks target skills beyond what students can do independently. Discussing strategies with their group members, writing complete solutions, and presenting their reasoning to the rest of the class, students develop their mathematical communication skills significantly. In recent years, the Team-Based Learning structure has been applied with much success to select Calculus I sections at Iowa State University. Quantitative data has shown that the TBL students performed better on the midterm and final calculus exams, providing higher quality explanations. A key component of the success of the Team-Based Learning method is student attitudes. To this end, a qualitative study was performed in the spring of 2018, examining the mathematical mindsets which influence the experiences and attitudes of students in a TBL classroom. In this

talk we will explain how the TBL structure was applied to the Calculus curriculum and share samples of rich mathematical tasks along with associated student work. We will also present the results of the quantitative and qualitative studies as well as ideas for future research.

**Marianna Bonanome** New York City College of Technology

#### Opening Gateways: Successful Activities and STEM Applications for Algebra and Trigonometry Courses

Opening Gateways to Completion is a 5 year Title V collaborative grant between the New York City College of Technology and the Borough of Manhattan Community College, CUNY. As part of the grant's activities, cohorts of full-time and part-time faculty grant participants from both campuses take part in an intensive professional development seminar, where they are exposed to active learning strategies, open digital pedagogies, flipped classroom techniques, games in the classroom, teaching with technology and much more. Join us as we present some of the exciting activities and STEM applications developed by our grant participants for their Algebra and Trigonometry classes.

## Part B: Thursday, August 2, 1:30–6:05 PM, Governor's Square 12, Plaza Building

**Tian Yu Yen** University of Colorado Denver

#### Building Community in the Classroom: Creating Classroom Culture and Establishing Community Norms

For many students, learning mathematics in the college and university setting can be extremely isolating: even when encouraged to collaborate, students may find it difficult to work together because of differing math backgrounds and/or anxiousness about being wrong in front of their peers. One strategy to create more productive collaboration between students is to spend time building community in the classroom. We share several activities and approaches to building community in the first few days of the semester including, a variation on Dana Ernst's "Setting the Stage" conversations, developing Community Norms, and Name Circles. Then we present some of the implications, successes, and failures of implementing community building in the classroom.

**Reza O. Abbasian** Texas Lutheran University
**Michael L. Czuchry** Texas Lutheran University
**John T. Sieben** Texas Lutheran University

#### Investigation of Inverted and Active Pedagogies in STEM Disciplines: A Preliminary Report A Preliminary Report

In this talk we will discuss preliminary results comparing student achievement between inverted (flipped) classrooms and traditional lecture formats in natural science and statistics classes at Texas Lutheran University. These results are part of a three-year NSF funded grant titled "Inverted and Active Learning Pedagogies (IALP) for Student Success" ( faculty investigators from biology, chemistry, physics, mathematics and statistics). We will provide a brief description of the goals of grant, the methodology used for the study including instruments that were developed to measure students' learning in inverted versus traditional instruction across multiple sections of the same course and across different disciplines. In addition, the methodology of the longitudinal study for examining student success in inverted and traditional courses across sequential courses will be provided. We will then present the results based on two semesters of data.

**Charlotte Knotts-Zides** Wofford College

#### Active Learning via Fill-in-the-blank Proofs in an Intro to Proofs course

Students in my Intro to Proofs class are required to submit proofs to homework or questions about proofs via an online journal assignment the night before class; homework problems that generate multiple concerns or demonstrate repeated mistaken reasoning form the basis of the active learning activity during class. Working in groups, students are presented with a version of the proof with blanks to fill in; these blanks may include definitions to complete or theorems to apply to the problem or they may be given a step and asked what definition, theorem or rule justifies this step. In this talk, I'll provide examples of the fill-in-the-blank proofs and discuss how to design and implement fill-in-the-blank activities. These proofs help students understand effective mathematical writing technique as well as make explicit the logical connections between each step of the proof; by working in groups, students benefit from discussions that result when there is disagreement about a step. This activity is adapted from a technique dubbed "Frames" in Elizabeth F. Barkley's "Student Engagement Techniques: A Handbook for College Faculty" .

**Jason Molitierno** Sacred Heart University

#### Projects Applying Linear Algebra to Calculus

Many math majors take Linear Algebra either directly after or towards the end of their calculus sequence. Students are often surprised at how little calculus there is in a linear algebra course. In this talk, I will discuss projects that I have given my Linear Algebra students that apply linear algebra to calculus. Some examples involve relating linear transformations to calculus, and applying linear algebra to differential equations. These projects help students understand that there truly is an interrelation between areas of math that, on the surface, seem quite different.

**Sarah A. Nelson** Lenoir-Rhyne University

### Embodied Activities: Engaging Students via Life Size Exploration

At the 2016 MAA MathFest, I was fortunate enough to participate in the Project NExT workshop led by Dr. Hortensia Soto (University of Northern Colorado) on embodied activities. By having students take on the role of the object(s) they are studying, embodied activities afford students the opportunity to experience those concepts and challenge students to develop a deeper understanding of the related definitions. My experiences have shown that these assignments promote student engagement, allow students to struggle productively within a safe environment, and support student success.In the Fall of 2017, I applied for a Faculty Micro Grant through Lenoir-Rhyne's Center for Teaching and Learning. With this award, I was able to design and implement various embodied activities into my Spring 2018 courses. In this talk, I will discuss materials used, how I adapted the geometry activities shared by Dr. Soto, and how I created my own activities for studying Venn Diagrams, the Cartesian Coordinate System, and graphing. I will also share how student feedback was utilized to make improvements on new activities and some important lessons learned.

**Kristen Sellke** Saint Mary's University of Minnesota

### Intentionally Integrating Prior Knowledge into Daily Lessons

In a first implementation of the flipped classroom pedagogy, students watched short videos covering new content before coming to class. Students in these classes indicated high levels of satisfaction with this preview of the upcoming class; it worked well for classes with shorter meeting periods where content could be broken down into smaller pieces. It was difficult to implement in classes with longer meeting periods or more involved content. To give students in these classes a preview of the upcoming class a different method was used. In this second method, students were given a set of questions or computations to complete in preparation for the next class. These questions involved no new content. Students were able to complete the preparation problems using only prior knowledge and skills. For example, in one preview activity students were asked to repeat the same computation with various initial conditions and look for patterns and make predictions based on the results. The outcomes of the pre-class work then guided the lesson and allowed the class to go deeper into the new material. Examples of preview assignments from all levels of the major curriculum, including calculus, linear algebra, differential equations and analysis will be shared.

**Gus Greivel** Colorado School of Mines
**Scott Strong** Colorado School of Mines

### A SCALE-UP Instructional Model for Multivariate Calculus

The department of Applied Mathematics and Statistics at the Colorado School of Mines (CSM) is exploring a variety of pedagogical strategies to (i) encourage active learning in multi-section core mathematics courses and (ii) make the content of these courses more engaging and relevant to our students. With support from the Office of Naval Research, we have recently developed a SCALE-UP (Studio) version of our Honors Calculus III and Honors Differential Equations courses sequence. In the studio, we introduce group activities focused challenging problems and concepts with a goal of stimulating discussion and reflection. We also support these activities with undergraduate learning assistants and Mathematica notebooks designed around the particulars of the assignment. This talk will discuss our Calculus III Honors course redesign, student feedback and work, lessons learned, additional goals related to our grant, and future plans for the instructional model.

**Sarah Wright** Fitchburg State University

### Sort The Sequences

In this talk we'll discuss the outline of a lesson plan for a class covering the introduction to Infinite Sequences in a second semester Calculus course. This plan fits into one 75 minute class and my own flipped classroom structure, but could be easily adjusted for other situations. Additionally, I hope to discuss ideas of other topics where similar ideas may also be successful.

**Daniel E. Otero** Xavier University
**Dominic Klyve** Central Washington University
**Nicholas A. Scoville** Ursinus College
**Diana White** University of Colorado Denver

### Instructors' Experiences using Primary Source Projects in mathematics classrooms

TRIUMPHS (TRansforming Instruction in Undergraduate Mathematics via Primary Historical Sources) is a five-year, NSF-supported project dedicated to designing, developing, and disseminating classroom modules for teaching college-level mathematics via primary historical sources, and to study their implementation in the classroom. The project, just completing its third year, is beginning to analyze data it has collected from dozens of instructors who have used these "Primary Source Projects" (PSPs) in their classrooms. In this talk, we offer some preliminary findings from instructors' responses to Pre- and Post-course surveys, and from Implementation Reports they completed related to the use of a PSP. We include information about the instructor/site tester demographics, their prior experience with history of mathematics and their use of primary sources in the undergraduate classroom, with a focus on instructors' perceptions on the impact of PSPs on their students and on their teaching.

**Nora Strasser** Friends University

### Engaging Students With Augmented Reality

Today's students use technology in almost everything they do. Many Math classes seem less engaging because of the lack of cutting edge technology. One way to improve student engagement while also improving learning is to embed augmented reality (AR) clips in the written word. Using HP Reveal, a teacher can quickly create interactive AR clips within both online resources and written resources. A photo of the instructor on a syllabus can be turned into a short video clip of the instructor welcoming students to class. An applied problem on a worksheet can include a video clip of the physical process. Problems can have embedded hints and animations. This talk will describe how these AR clips can be created and how they have been incorporated into classroom assignments. The impact on student learning will be discussed.

**Mile Krajcevski** University of South Florida

### Drawing-to-Learn activity as a cognitive tool in undergraduate mathematics

We provide a framework for so-called drawing-to-learn activity as a cognitive tool that helps students model abstract mathematical notions into concrete visual objects by drawing. Given proper guidance on how to visualize by drawing, students do not only interpret a visualization of a mathematical object created for them but they also acquire new mathematical information from the drawing. We illustrate this approach with examples from undergraduate linear algebra course and point to the potential this cognitive method has in teaching undergraduate mathematics.

**Carrie Muir** Whatcom Community College

### Making Connections with Card Sorts

Card sorts are simple activities to help spark student engagement, which can be used for a wide variety of math topics and course levels. They can be structured as individual student activities, small group activities, or full class activities; can be done with physical cards or online; and can be used to review material, as formative assessment, or as summative assessment. This session will include a sample of creating a card sort for linear equations, revising the activity after initial class use, and using the activity for multiple purposes in multiple courses. Samples of additional card sort topics and structures will also be presented.

**Lisa Bromberg** United States Military Academy - West Point
**Kayla Blyman** United States Military Academy - West Point
**Kristin Arney** United States Military Academy - West Point

### Transforming Mathematics Assessments to Drive Better Learning

In mathematics classes, it is common to find students asking when they are going to use the material in the real world. We are taking steps towards removing the sterile facade that often plagues a mathematics classroom during exams by assessing students on their ability to apply course material in a more realistic environment with collaboration and technology available to assist them. This method of assessment is being piloted in *Mathematical Modeling and Introduction to Calculus*. This course, rooted in mathematical modeling with discrete dynamical systems, is the first mathematics course taken by approximately 900 students at the United States Military Academy. This method entails weekly assessments in place of major exams. The assessments consist of three parts: a night before read-ahead focused on a new application, an in-class individual portion where students respond to short answer questions, and an in-class group portion where groups of 3-4 students provide team responses to the same questions after discussion, learning, and consensus. We believe this will result in better attainment of higher order learning goals, better preparation for professional collaboration, increased technology skills, and more creative excellence. Sample assessments are provided.

**Mary A. Nelson** George Mason University

### Oral Assessments: Helping Students Make Connections

This paper will discuss the advantages of formative oral assessments (orals) in improving student understanding. These assessments are voluntary and ungraded. Small groups of 6-10 students meet with a facilitator for an hour and discuss the underlying ideas that will be tested on an upcoming written exam. Students are asked to make connections and negotiate meaning with their peers and the facilitator. Emphasis is on how and why. Students often are asked to draw graphs and figures to explain their reasoning. These assessments allow students to be more metacognitive about their learning and allow facilitators to determine "where students are" . Misconceptions can be addressed, and instructors can identify concepts that need further explanation in class. Data show that students of all ability levels benefit from orals. Students have been shown to attend class more often and complete more homework in classes where orals are offered, and orals have been repeatedly correlated to about 1 letter grade improvement in final course grades. At one major university where 9 years of data show an average 31% failure rate for Calculus I before the introduction of orals, failure rates dropped to an average of 22% for the next five years with the addition of orals. The author will describe other changes made in the mathematics department to improve retention. These include the introduction of a two-semester Calculus I course, the addition of learning assistants and the creation of a unique course for students earning a 3 on the Calculus AP test.

# Fostering Undergraduate Interdisciplinarity

## Friday, August 3, 1:30–6:25 PM, Governor's Square 15, Plaza Building

**Organizers: Amanda I. Beecher** Ramapo College of New Jersey
**Chris Arney** United States Military Academy at West Point

Mathematics is one educational tool to develop complex problem solvers that are needed to address many of the largest and most challenging problems in society, which are often interdisciplinary. This session invites speakers to present their efforts to foster interdisciplinary work by undergraduates within courses or outside the classroom.

**Kasie Farlow** Dominican College

### Interdisciplinary Work At a Small Institution

There are many challenges and benefits to being in a small institution or department. Taking advantage of interdisciplinary opportunities exposes students to other disciplines and the many applications of mathematics. In this talk I will discuss encouraging students to participate in activities such as the Interdisciplinary Contest in Modeling and the Mathematical Contest in Modeling as well as examples of interdisciplinary collaboration among colleagues at a small school.

**Sijia Fan** Skidmore College
**Ran Tao** Skidmore College
**Kaifeng Yang** Skidmore College

### How does climate change influence regional instability?

This paper was a contest submission for Interdisciplinary Contest in Modelling (ICM) in 2018 and won an Outstanding Paper Award, COMAP Award, and INFORMS Award. Our team built a mathematical framework to identify the status of a country, motivated to measure its stability quantitatively and more accurately. In our paper, we categorize countries into three states in terms of its fragility: stable state, vulnerable state, and fragile state. We consider different factors including economy, politics, demographics and climate change. Based on the logistic growth model, we create a dynamic system that describes the current fragility of a country and predicts the equilibrium states the country would approach overtime. Moreover, we utilize a bifurcation plot to depict the turning points between three states of a country. We then apply our framework in real-life case analyses with three selected countries, Syria, Cuba and New Zealand, each in fragile, vulnerable, and stable state, respectively. The results show that New Zealand is steadily reaching equilibrium and reveals that Syria and Cuba's current states are significantly impacted by negative climate changes. However, after calibrating the climate variable to the world average, we find that both Syria and Cuba would approach vulnerable and stable state, respectively.

**Victor Piercey** Ferris State University

### Collaborating with Partner Disciplines to Develop Interdisciplinary Simulations, Case Studies, and Inquiry-Based Activities in Quantitative Reasoning

At Ferris State University, we have developed quantitative reasoning courses to serve as a hub to connect mathematics with selected partner disciplines. We have worked with business, health professions, and social work to develop an interdisciplinary general education math sequence that we call Quantitative Reasoning for Professionals. In this talk, I will describe how faculty in each of these disciplines collaborated to develop inquiry-based explorations, case studies, and role-playing simulations to guide students to thinking deeper about mathematics, integrate mathematics into their professional and personal worldviews, and develop a sense of problem solving.

**Michelle L. Isenhour** Naval Postgraduate School
**Ralucca Gera** Naval Postgraduate School

### Incorporating Disciplinary Knowledge through Adaptive Learning Modules

In this talk we discuss the potential to incorporate varying levels of disciplinary knowledge into innovative adaptive learning modules that use a variety of delivery methods, to include demos, videos, interactive applications, TED talks, and webinars. The instructor facilitates the learning experience by presenting the state of the art of the subject, together with how it can be used, while guiding the students through a web of the aforementioned demos and videos. We introduce our vision and provide a baseline template that we believe all adaptive learning modules must follow in order to provide consistent quality instruction and facilitate learning. Lastly, we'll demonstrate how instructors are encouraged to complement this basal structure with innovative methods of incorporating disciplinary knowledge into the modules at varying levels of breadth and depth. While we briefly discuss our "student scaffolding" mathematical modules, the focus of this talk is on our "state-of-art" as well as our "technical" modules which we believe will foster an interdisciplinary interest in the student. The "student scaffolding" modules are short 15-30 minute modules which teach the necessary mathematical skills (on an individualized as-needed basis) required for the follow-on "state-of-the-art" and "technical modules" . The "state-of-the-art" modules are 1-2 hour modules which provide an overview of main topics from the discipline, whereas the "technical" modules are 1 hour modules providing the details of how the topics covered in the "state-of-the-art" modules are implemented in the discipline. This approach will not only enhance the student's mathematical abilities but also exposes the student to applications across numerous disciplines.

**Sarah Cobb** Midwestern State University
**Jeff Hood** Midwestern State University
**Peter Fields** Midwestern State University

### Connecting Disciplines Using Science Fiction

In this talk, I will discuss the integration of mathematics into a general education writing course using Andy Weir's novel *The Martian*. The course used situations in the novel to introduce students with a variety of majors to mathematical ideas including probability, modeling, and measuring power. This talk will describe the structure of the course as well as some interesting mathematical insights into the novel.

**Mark Rasmussen** Siena Heights University

### Fauré or 4A: A Foray into the Math of Music

While the fact that ties exist between mathematics and music is well known, knowledge of the details of these connections is not as ubiquitous. We will be looking at the development of a course designed to explore these links and use music and musical ideas as a medium convey mathematical concepts, increasing students understanding of both topics. This talk will look at various aspects of course construction including choice of topics and methods of assessment.

**Kelly Black** University of Georgia
**Guangming Yao** Clarkson University
**Michael Ramsdell** Clarkson University
**Craig Wiegert** University of Georgia

### Coordinated Calculus and Physics

The calculus class is an important part of our first year students' experience, and it is the gateway to most of their STEM coursework. The introductory physics class carries the same weight and impact on our students' success as well. Unfortunately, these two vital courses are taught separately despite close historic ties and closely related topics. To compound matters, the courses are generally offered in ways that do not necessarily cater to our students' needs. To address this issue we have developed a method to categorize student needs based on their scores on basic mathematical skills and their conceptual understanding of basic mechanics concepts. Students are given options for a program to better suit their needs. In this discussion we focus on an approach to the calculus sequence for students who face challenges with their basic mathematics skills but have demonstrated a firm grounding in their understanding of physics concepts. The calculus course is tightly linked with the first year mechanics course, and the we take advantage of their deep intuition to develop their understanding of mathematics concepts.

**Stanley Florkowski** III, United States Military Academy, Cornwall, NY
**Steven Morse** United States Military Academy

### The Moose and Wolves Project: UnitingDifferential Equations, Vector Calculus, and Population Ecology in a Case Studyof the Isle Royale National Park

We present an interdisciplinary undergraduate project based on an ongoing crisis of ecological balance in the Isle Royale National Park, used as a capstone in the freshman advanced mathematics program at the U.S. Military Academy. Students use a predator-prey system to model population trajectories of moose and wolf on the island, with connections to vector calculus and parameter estimation using constrained multivariable optimization. Strong, interdisciplinary emphasis is placed on the students being able to defend the mathematical analysis they perform to stakeholders with non-technical backgrounds. We discuss the project and present feedback from students and faculty on challenges and positive learning outcomes.

**Nathan Carter** Bentley University
**Kenneth G. Monks** University of Scranton

### Lurch validates Plato: an application of proof verification software to philosophy

Lurch is an open-source word processor that can check the steps in students' mathematical proofs. Users write in a natural language, but mark portions of a document as meaningful, so the software can distinguish content for human readers from content it should analyze. While the primary target audience consists of students in a first Introduction to Proof course for undergraduate math majors, there is no reason why it cannot be used to validate reasoning in other areas. This talk reports on a student's delightfully unexpected application of Lurch to a graduate assignment in philosophy, where Lurch was able to validate an argument in Plato.

**Adeline R. Jacobsen** University of Washington-Tacoma
**Connor Louis Myers** University of Washington-Tacoma

#### Where To Draw The Line: Metrics of Gerrymandering

Where To Draw The Line Gerrymandering Metrics Adeline Jacobsen, Connor Louis Myers 2018 Math Fest April 30, 2018 Gerrymandering is the manipulation of congressional districts to increase the probability of a desired outcome. In our presentation we will explore two different metrics for measuring gerrymandering,*The Efficiency Gap* and *The Polsby-Popper Test.* The Efficiency Gap, which takes voting data from a prior election and uses the concept of a wasted votes to try and indicate if a district has been gerrymandered, and The Polsby-Poppe Test is a measure of compact- ness of congressional district, to see if the shape of the district indicates any manipulation. We will explore these metrics, learn how to compute them and some of the undesirable properties.

**Terry Griffin** Midwestern State Univeristy
**Marcos Lopez** Midwestern State Univeristy

#### Building a Research Group on an Empty Lot

This talk will cover how our small one semester project grew into an ongoing research group between math and computer science students. Our research group started off with two students looking into how find better parking at our university. At first, we were using simple averages and interpolation in a few lines of code. We now are a steady group with 8 undergraduates that work on front and back end app development, machine learning, and networking.

**Jeong-Mi Yoon** UH-Downtown
**Weining Feng** UH-Downtown

#### An Interdisciplinary Undergraduate Research Project in Compressor Surge Modeling

The centrifugal compressors are an essential major equipment industrial production systems such as gas production, petroleum-chemical industry, power generation and etc. Surge happens when a compressor operation drifted into the unstable region which may cause significant damage of the compressor. The safe and efficient operation of compressors has been presenting significant technical challenges Two undergraduate math students were recruited at the beginning of the spring of 2018, under the supervision of a Math and an Engineering professors. Initially students were been introduced to the fundamentals of Differential Equations, Thermodynamics, and MATLAB/SIMULINK. Then an existing compressor surge model were introduced to them. As the first step they were tasked to regenerate and interpret the compressor characteristic map and surge points. In future semesters we will explore the solution of the dynamic model numerically and analytically. The numerical solution will be implemented on a software platform such as MATLAB/ SIMULINK and the analytic solution will be answered by the stability analysis. All these interdisciplinary efforts have been carried out in the form of voluntary research work for both faculty members and students. This is a significant initiative in promoting high impact interdisciplinary learning and research experience for undergraduate students and connect them to their senior projects in future.

**John Haga** Wentworth Institute of Technology

#### Catastrophe Modeling: A Case Study in Vocational Curriculum

Students enrolled in the course “Catastrophe Modeling” were tasked with modeling insured loss due to rare but catastrophic disasters. The inherent difficulty is that data driven methods fail due to infrequent occurrence. Monte Carlo methods were implemented to simulate the occurrence of a hazard (flood, wildfire, et al), the exposure of an asset to the hazard, and the insured loss due to this exposure. Modeling the hazard requires an understanding of the scientific principles that guide dynamics and intensity, and analysis of local geographic data. Simulating exposure to the hazard requires an analysis of asset characteristics (building materials, structure, etc). Incorporating financial loss involves statistical analysis of historical claims data. The project was conceived with input from members of industry; the techniques used were approximately industry standard, and the course was by nature highly interdisciplinary.

**Mami Wentworth** Wentworth Institute of Technology

#### What’s the damage? Modeling of cholera dynamics to compute the cost of insurance.

A course on Catastrophe Modeling was offered recently, in which student learned to model three types of events (wildfire, flood and disease) and computed expected cost of damage in case of catastrophic events. Students combined knowledge from multiple fields within mathematics (numerical analysis, ODE’s, Linear Algebra) as well as other disciplines (computer science, civil engineering, finance) to complete their projects. Additionally, the course was advised by external collaborators from a risk analysis firm, and both faculty and student received feedback from the representatives throughout the semester. In this talk, I will present the process of developing our interdisciplinary, project-based course with external collaborators. I will also present initial results from the course, with the focus on the modeling of cholera disease in Haiti.

**Edward W. Swim** Sam Houston State University
**John G. Alford** Sam Houston State University

**Save the cranes! Mathematical modeling within an environmental conservation effort.**

The importance of integrative modeling and computational solutions within the biological sciences motivated us to create a long-term undergraduate research program in environmental mathematics. Our research, in collaboration with faculty from our biology department, is rooted in a study of ecosystem dynamics in the coastal marshes of the Aransas National Wildlife Refuge (ANWR). Recent exceptional drought, hurricanes, oil spills, and other severe disturbance events have the potential for a significant and long-term impact in this coastal system. This is especially true considering the over 400 species of coastal migratory birds, including the endangered Whooping Crane, which winters exclusively in the coastal wetlands surrounding the ANWR. The presence of the Whooping Crane and the heterogeneous nature of the ANWR ecosystem have provided multiple sources of inspiration for the creation of mathematical models to assist in ongoing local conservation efforts. Here we highlight some of the outstanding results obtained since our program began in 2013 and discuss the elements that were essential for success in creating and sustaining the program.

# Great Circles, Great Problems

## Thursday, August 2, 1:30–5:25 PM, Governor's Square 15, Plaza Building

**Organizers: Amanda Matson** Clarke University
**Diana White** National Association of Math Circles

Math Circles are outreach programs for K12 teachers and students, often led by university-based mathematicians, which focus on providing authentic mathematical experiences –where participants enjoy and engage with mathematics as a lively discipline of inquiry, conjecturing, and problem solving. Presenters will share mathematical problems and activities that can lead to hours of exploration by the curious.

**Sarah Trebat-Leder** Art of Problem Solving

**Polyominoes and Blokus**

The pieces for the game Blokus consist of all possible polyominoes with up to 5 cells. In this talk, we'll discuss some mathematical explorations inspired by Blokus that were done with a group of middle school students at Emory Math Circle.

**Amanda Katharine Serenevy** Riverbend Community Math Center

**Queen Dido Problems**

Queen Dido and her followers had to flee from Tyre, because her brother was plotting to kill them. Dido led her followers to northern Africa (modern-day Tunisia) where she negotiated with the Berber King Iarbas for a piece of land where they could all live. King Iarbas agreed to give her as much land as she could encompass within an ox's hide. Dido cleverly cut the ox's hide into thin strips and stitched them together to make a very long string. Now Dido wants to enclose as much land for her followers as possible using that strip. This Math Circle topic includes material from Jakob Steiner, Alan Siegel, and Huang Tran.

**Mahmud Akelbek** Weber State University

**Explore Transformations through Anamorphosis and 3D Art**

Transformations are widely used in mathematics and other disciplines. Broad applications of different transformations can be found in computer graphics as well. In this talk we present hands-on activities related to anamorphosis and three-dimensional art. Both of them are related to transformations. We will also discuss transformations between different coordinate systems.

**Sarah Bryant** Dickinson College
**Lance Bryant** Shippensburg University

**Positive *Net* Results: Folding and Unfolding**

We will share one of our favorite problems explored in the Shippensburg Area Math Circle for $4^{th}$ and $5^{th}$ graders. We start with odd (but familiar-looking) flat shapes, then quickly realize these are unfolded versions of 3-dimensional objects. Before folding them up we sort according to identifiable properties and then test our ability to identify objects by their unfolded versions (nets). We then try to identify all the possible nets for a cube and then even try to mentally fold nets so they match decorated cubes. Lastly, we play with an unfolded Rubik's cube (via the Rubik's Cube Wiki) and are left wondering: does every polyhedron have a net? Lots of visualization and play make this one of our very favorite problems!

**Erica Bajo Calderon** University Of Central Oklahoma

### Catapult Planning and Development Activity at the Central Oklahoma Math Circle

The Central Oklahoma Math Circle is a partnership between US Grant High School in Oklahoma City and the University of Central Oklahoma. Creativity, ingenuity, open-mindedness, persistence, and communication are key factors in our Catapult Planning and Development Activity. In this talk we will present the guidelines and limited instructions given to students to help them create a catapult and complete certain tasks. This activity provides an experience in which the students are required to think critically, manage time wisely, and work together in order to be successful. We will share the different approaches and models the students used along with any adjustments that were made. Finally, we will present variations to this activity.

**Mark Koester** MSU Denver

### Pythagorean Triples: Connections Between Algebra and Geometry

In this session I will share the experience of my facilitating a session of a Math Teacher Circle using a Primary Source Historical Project. These are inquiry-based projects from the TRIUMPHS project that are designed using primary documents. In this project, Pythagorean Triples were generated using two different geometric methods. This task is especially well-suited for exploration and discussion as it connects a typically algebraic idea to a geometric interpretation. Discussion was robust about how the geometric representation supported algebraic thinking and how the algebraic symbolism connected to the geometry. We ended the session with discussion about implications for teaching.

**Rebin A. Muhammad** Ohio University

### Using Paper Folding to Create Islamic Geometric Pattern

We have already used Islamic Geometric Pattern (IGP) in different math circle sessions. We used different methods to create the patterns, from compass-straightedge to point-line method construction. This time we will use paper folding to create them. The advantage of this method is that it is safer than working with a compass and it is also more practical. In addition, we can ask more concrete geometric questions and questions about angles of polygons. We can then physically answer those questions through paper folding. A typical session starts with providing students with a guide to create some types of regular polygons from a square of paper (like an equilateral triangle, a hexagon, etc.) and then we continue with using these shapes to create some of the patterns we give them. Later we will ask the students to create their own patterns under some conditions (number of different shapes, types of induced shape).

**Na Yu** Lawrence Technological University
**Ruth Favro** Lawrence Technological University

### Math Circle at Racquet Up Detroit

Racquet Up Detroit is an out-of-school youth development program with the game of squash, an academic enrichment program, tutoring, community service, and mentoring. Currently there are almost 100 students in the program ranging from fifth grade through high school. The goal of our math circles is to offer students who would not get opportunities for math enrichment the chance to discover the joy of problem-solving and the beauty of math in an environment where they would be supported by the group, group leaders, parents and teachers. This presentation will share the activities we have done and the stories of our students.

**James C. Taylor** Math Circles Collaborative of New Mexico

### Competitive Constructions: Polyhedra, MESA, and Intuition-first

A chance invitation to design a brief, competitive-format math activity from our regional MESA coordinator led to a new take on the old "Space Chips" math circle activity about constructing and understanding polyhedra. In this session I'll discuss my experiences with roughly 250 secondary level students–in teams–at two MESA festivals, and my thoughts about the value of using this sort of hurried introductory phase to a math circle as an intuition and team building exercise setting the stage for a deeper investigation.

**Brianna Donaldson** American Institute of Mathematics

### The Community Alliance for Mathematics

The Community Alliance for Mathematics (CAM) is a new collaborative endeavor with partners including the American Institute of Mathematics, the Mathematical Sciences Research Institute, the MAA, the Math Teachers' Circle Network, the National Association of Math Circles, the Julia Robinson Mathematics Festival, the Alliance of Indigenous Math Circles, the Indigenous Math Circle Communities, and the Mathematicians of Color Alliance. The vision of CAM is for students, teachers, and families of every color, gender, and income level, in every ZIP code, to have meaningful access to equitable out-of-school spaces for doing mathematics. CAM will foster a nationwide web of community engagement with mathematics as an active, creative, and living discipline that belongs to all. By increasing collaboration among partner organizations and growing sustainable regional infrastructure that intentionally focuses on reaching underrepresented groups through partnerships between teachers and mathematicians, CAM will create a densely interconnected web of coordinated Mathematical Communities: out-of-school mathematics spaces that include one or more Teacher-Mathematician Partnerships, Math Student Circles, Math Teachers' Circles, and Julia Robinson Mathematics Festivals. By transforming the mathematical identities and motivation to persist in mathematics of underrepresented students, their teachers, and their families, CAM has the potential to transform the face of mathematics.

# Inquiry-Based Learning and Teaching

## Part A: Friday, August 3, 9:30 AM–12:25 PM, Governor's Square 14, Plaza Building

**Organizers: Brian Katz** Augustana College
**Eric Kahn** Bloomsburg University
**Victor Piercey** Ferris State University
**Candice Price** University of San Diego
**Xiao Xiao** Utica College
**Amanda H. Matson** Clarke University
**Mindy Capaldi** Valparaiso University
**Kayla Dwelle** Ouachita Baptist University
**Phong Le** Goucher College

Inquiry-Based Learning approaches seek to transform students from consumers to producers of mathematics. Inquiry-based methods aim to help students develop a deep understanding of mathematical concepts and the processes of doing mathematics by putting those students in direct contact with mathematical phenomena, questions, and communities. This session invites scholarly presentations on the use of inquiry-based methods for teaching and learning.

**Jessica Williams** Converse College

### Transitioning from Lecture to IBL

Encouraged by research supporting the effectiveness of inquiry-based learning in undergraduate proof-based courses, undergraduate Number Theory and Modern Geometry courses at a small college were re-designed to be taught in IBL fashion. The courses were previously taught in a primarily lecture format and based upon a selected textbook. This talk will describe a general method for transitioning proof-based courses from lecture style to IBL style while still using a central text. Structure of the overall courses, strategies for designing problem sets, breakdowns of typical class periods, and modifications made will be discussed. Course-specific materials and templates will be shared. The focus will be a practical approach to efficiently re-designing a course to be taught in inquiry-based learning style for the first time.

**Kristen Pueschel** Penn State New Kensington

### Successes (and failures) from a first attempt at inquiry

In Fall 2017, I used the Inquiry Oriented Linear Algebra materials in my introductory matrices class. These materials have the benefit that they are well documented, with substantial instructor resources such as past student work and important discussion points. This makes these materials a friendly starting point for a first-time IBLer like me. In this talk, I will discuss some successful in-class and out-of class activities from this course, as well as some potential pitfalls to avoid.

**Robin Cruz** The College of Idaho

### IBL in Linear Algebra: Same Theory, More Practice

Ten years ago, I taught a Linear Algebra course using notes developed by David M. Clark. The students in the class were mostly math majors, and by the end of the semester, I felt encouraged to continue with an IBL format in the class. As time went by, the percentage of math majors to other majors–such as Physics and Computer Science majors–taking the course decreased. Accordingly, the course evolved. My Linear Algebra course this year was a satisfying blend of theory and computation. I will describe the strategies I used to get students who don't think they like doing mathematical proofs to work at honing their proof writing and critical listening skills.

**Jeb Collins** University of Mary Washington

### Productive Failure of an IBL Proofs Course

Teaching a lecture-less inquiry based learning course, what is sometimes called "full" IBL, for the first time can be intimidating for one who has never done it. Much like our students, the reason for the intimidation is often fear of failure. The honest truth is that "failure" is certainly a possibility, but also like our students, we must be willing to learn from that failure and try again. This talk will outline the first IBL proofs course I taught, which failed spectacularly. I will go over how the course was run, why I think it failed, but more importantly, the lessons I learned from teaching this course. My hope is that these lessons will help other first-timers not make the same mistakes. However, I also hope this will encourage those who have not had an ideal experience to try again, know they are not alone, and learn from my mistakes.

**Mike Janssen** Dordt College

#### Promoting Inquiry with Recreational Problems in a Liberal Arts Math Course

A frequent goal of the so-called liberal arts mathematics course is that students have an experience common to most professional practitioners of mathematics: the joy and delight of inquiring after an interesting problem. We will discuss the use of recreational mathematics puzzles as a regular feature of such a course as a tool to promote mathematical habits of mind such as persistence and inquiry. Student stories and reactions will be shared, as well as suggested problems and assessments.

**Violeta Vasilevska** Utah Valley University

#### Using IBL: Daily Group Work in Calculus Classes

This presentation will highlight the implementation of a modified Inquiry-Based Learning method (through group work) in Calculus I and II classes. Namely, in these classes, students are provided with lecture worksheets that are based on IBL type notes. After the students are guided to discuss new concepts and ideas, they work in groups to apply those concepts on carefully designed problems (contained in the provided lecture worksheets). Then each group chooses one team member who will be the 'presenter' for their solution/idea to the class and the class discusses the solution/idea presented. In this talk, the structure of the in-class group work, the benefits to students (and teacher), and the students' feedback on the in-class group work/worksheets will be discussed.

**Aliza Steurer** Dominican University

#### Comparing Inquiry-Based Learning to Parenting

While learning how to teach using inquiry-based learning (IBL) methods, the presenter noticed similarities that these techniques have with parenting. In this talk, we will examine parallel aspects between IBL and parenting, including focusing on the process of solving a problem instead of on the final result, making mistakes and learning from them, and deconstructing tasks into smaller, more manageable ones. We'll also discuss instructor- student and parent-child interactions, including responding to difficult questions in the moment, sensitivity to body language, and active listening. The comparison between parenting and IBL appears to be a perspective not found in the current literature. The aim is to illuminate aspects of and better understand IBL.

**Mariah Birgen** Wartburg College

#### Tips and Tricks for Tracking a Student Centered Class

Teaching an IBL mathematics class can often feel like transitioning from trying to herd cats to sitting in the kitten room and watching appreciating watching them crawl all over each other. When it works, this brings a warm and fuzzy feeling to your heart, but then you realize that you need to keep track of all this chaos and have something for the assessment gurus at the end of the term. Fear not, this is possible to do and in such a way that your students will become more engaged and not less. The trick is to include discussion tracking as part of the responsibility of the student and not solely that of the teacher. This talk will go over a variety of successful and not-so-successful ways to include students in the tasks of classroom management and discussion tracking. I will give you at least one, concrete method that you could use in your class and a variety of things you could do to personalize the technique for your personality. Finally, I will explain how this works to create a more student-owned learning space where the emphasis is on mathematics and learning and not on grades.

**Anne Sinko** College of St. Benedict/St. John's University

#### Specifications Grading in an IBL Classroom

One of the advantages for students in an IBL classroom is that students are given time to encounter and develop their understanding of the material. Traditional grading schemes, however, often reward students who quickly develop an understanding of the material and punish those who need time to digest new ideas (and how these new ideas relate to others). Furthermore, traditional grading schemes disincentive risk taking, an important component in my successful IBL courses. In this talk, I will discuss how I decided specifications grading would alleviate the challenges I faced with traditional grading schemes and provide examples of specifications grading in two IBL courses - a 100-level introductory probability and statistics course taught in a flipped style and an upper division linear algebra course taught in a Modified Moore Method style.

## Part B: Friday, August 3, 1:30–5:45 PM, Governor's Square 14, Plaza Building

**Nathaniel Miller** University of Northern Colorado

### An Online IBL Geometry Class

During the summers of 2015 and 2017, I developed and taught an online IBL class for Master's students who were in-service secondary teachers. The class met synchronously via videoconferencing software for 3 hours a day over the course of 16 days. The class time was divided between two strands. In one strand, students worked on a problem sequence from David Clark's IBL textbook "Euclidean Geometry: A Guided Inquiry Approach," presenting their solutions to the class on a daily basis, just as they would have in a traditional classroom. In the other strand, the students worked on inquiry projects in groups using open-source applications like GeoGebra and Spherical Easel. In this talk, I will discuss the course, how it was structured, and how IBL teaching methods can be successfully adapted to online classes.

**Melissa Riley** University of Nebraska at Omaha
**Michael E. Matthews** University of Nebraska at Omaha
**Dora Matache** University of Nebraska at Omaha

### Experience of a Noyce-student learning assistant in an Inquiry Based Learning class

This report refers to an undergraduate course called introduction to abstract mathematics at the University of Nebraska at Omaha. During the academic year 2017-2018, undergraduate, mathematics student One of the authors was a Noyce-student learning assistant for the Inquiry Based Learning (IBL) section of the course. We will report the results of a research project that focused on the impact of the class on students learning and perception of IBL. Additionally, special focus is placed on the learning assistant's own experiences. She assisted the faculty-in-charge with all aspects of the course. These included: materials preparation, class organization, teamwork, class leading, presentations, and tutoring. This presentation shall address some examples of how the IBL approach can be used in this type of class including: the structure of the course, the activities and tasks performed by the students, learning assistant, and faculty, the assessment of the student work.

**Gina Monks** Penn State Hazleton

### Integrating a Learning Assistant Program with a Dedicated Learning Center

This talk describes how we implemented a Learning Assistant Program at a small branch campus. In addition, it explores how we integrated the program with current student support services to best help our students succeed and what impact we found the program has on the students and the Learning Assistant.

**Chris Oehrlein** Oklahoma City Community College

### Sneaking IBL into College Prep Intermediate Algebra through 35-Minute Activities

College Prep Math at OCCC is designed with team-taught lecture, computer lab, and small group time. Students rotate through the stations in 35-minute periods. The courses are extremely structured, and the only rotation in which individual instructors have leeway to choose activities for their particular section of students is the small group time. Sometimes, if available, a lab instructor or partner professor will assist in small group. Most of the small group lessons and activities are collaborative practice of skills and applications introduced during lecture. Occasionally in the Intermediate Algebra-level course, the activities preview the modeling skills to which students will be introduced in College Algebra. This instructor decided to use his small group time as IBL time, introducing topics before lecture and using applications, data, and formula-skills from other disciplines to push his Intermediate Algebra students beyond what they considered mathematically of themselves. He will discuss the challenges, successes, and failures of his attempts, as well as his vision for the co-requisite models in College Algebra and Pre-Calculus that are a year away from being implemented.

**Betty Love** University of Nebraska - Omaha
**Victor Winter** University of Nebraska - Omaha
**Michael Matthews** University of Nebraska - Omaha
**Michelle Friend** University of Nebraska - Omaha
**Angie Hodge** Northern Arizona University

### Engaging Students in Algebraic Thinking by Pairing Coding with Active Learning Strategies

In this presentation, we will share how one university is conducting an NSF-funded pilot study in which the university algebra requirement was broadened to include a more hands-on approach to algebraic thinking. We will discuss how students can learn algebra in a new and creative way by pairing coding with active learning strategies. Preliminary results show that students are engaged with the material and able to demonstrate deep algebraic thinking.

**Gary A. Olson** University of Colorado Denver
**Heather Johnson** University of Colorado Denver
**Jeremiah Kalir** University of Colorado Denver

#### Implementing Desmos Techtivities to Promote Students' Covariational Reasoning

In this session, we will share five Desmos Techtivities designed in collaboration with Dan Meyer for use in a College Algebra classroom. These Techtivities link animations with dynamic graphs in innovative and exciting ways. Through interaction with these Techtivities, students form and interpret relationships between attributes capable of varying and possible to measure. We will also share ideas on how to integrate these Techtivities within an existing College Algebra curriculum in order to help students construct a more robust understanding of function.

**Sijia Li** Beijing National Day School
**Galit Eizman** Harvard University

#### The Effect of High School Reform on Students' Mathematical Achievements: Evidence from China

In recent years, there was a change in mindset among educators in China, who started to reform the education system to more westernized, individualized and dynamic, and as such- adapted to students' personal needs. From elementary to high-school education, there were changes in classroom practices, from a given schedule of courses and a traditional lecturing style to course selection options, students' transition between classes and various in-class activities. In this paper, we examine the relationship between this reform and high-school students' academic performances. The main assumption is that there is positive correlation between course selection system and the improvement in students' academic strength, particularly in subjects as mathematics. When students are allowed to select courses, they tend to make informed and careful decisions based on the course descriptions, their academic levels, and their preferred learning styles. They will be given the most suitable and relevant teaching materials and contents. The data used in this paper collected from one of the most prestigious and competitive high-schools in Beijing, China. On the sample of 2,349 high-school students whose graduation years range from the year 2012 to 2017, we examine how the reform of education system in 2011 influenced students' academic performances. Using panel data approach and Gaokao exam scores as a proxy, we are also able to explore causality. Our preliminary results show a positive and highly significant effect of course selection system on high-school students' academic performances, particularly in mathematics. These results imply that individualized and dynamic education systems are more likely to improve students' academic achievements.

**Julia St. Goar** Merrimack College
**Yvonne Lai** University of Nebraska-Lincoln

#### The Effects of Mathematical Mindset on the Future Implementation of Inquiry-Based Learning Methods by Pre-Service Elementary Teachers

Through the use of sources like Sybilla Beckmann's text among others, inquiry-based learning has made its way into pre-service elementary teachers' classes. For instance, at Merrimack College, the presenter has developed a second-semester math content course for pre-service elementary teachers incorporating inquiry-based learning. In addition, the course involves out-of-class problem solving and writing, as well as in-class written reflections on problem solving and hypothetical teaching scenarios. The question arises, is this inquiry-based approach affecting the way the pre-service teachers plan to teach their own future classes? Given the effectiveness of these methods, we strive for some of these techniques to be utilized in their future teaching, modified for appropriate grade-levels. To gain insight on this question, the presenter gave in-class written-reflections to pre-service elementary teachers, and we analyzed their responses to hypothetical teaching scenarios at the beginning and end of the semester. The results varied with some pre-service teachers incorporating increased student independence and active learning, and others whose outcomes were unchanged. The authors carefully inspected the students' written reflections to better understand why the outcomes varied between students. Additionally, for when outcomes did change, authors investigated which assignments or aspects of the course caused the greatest impact on how they conceived of their future teaching.

**Annie Selden** New Mexico State University
**John Selden** New Mexico State University

#### Knowing One's Goals for an IBL Proofs Course

In addition to having students become better at proving theorems, there can be many other goals for an IBL proofs course. In this presentation, we describe the mathematical and cognitive, as well as the psychological and affective, goals that we have for our *Understanding and Constructing Proofs* IBL course. We believe that most of these goals were originally implicit, but that it was a useful exercise to make them explicit. An example of a mathematical and cognitive goal we have is: *Students should have a variety of content-related proving experiences.* An example of a psychological and affective goal we have is: *Students should develop persistence and their capacity for searching and exploring.* To date, we have delineated and described 13 such goals.

**Theron J. Hitchman** University of Northern Iowa

### Bits of IBL: Modules in the Journal of Inquiry Based Learning in Mathematics

The Journal of Inquiry Based Learning in Mathematics (JIBLM) has served as a repository for IBL course notes for over a decade, giving IBL practitioners a way to share their work on course materials as a practical scholarship of teaching. The journal has focused on complete, semester-long courses so far. We advertise an expansion of the mission of JIBLM to include shorter units, that we call Modules; share a vision for what this new section of the journal can be, especially in conjunction with a new section of Reviews; and invite the participation of the IBL community as both authors and referees for these new efforts.

**Britney Hopkins** University of Central Oklahoma
**Jake Khoury** Virginia Commonwealth University

### Student Critique as an Inquiry-Based Practice

In this talk we describe a technique of using peer review of student work in a semi-flipped classroom environment to enhance student understanding in mathematics and composition courses. By recognizing shared learning objectives for formal arguments in sophomore level composition, introductory proof-writing, and calculus courses, we designed an inquiry-based collaborative activity that prompted students–both in small groups and as a class–to identify, apply, and critique key elements in constructing valid arguments and supporting those arguments in both disciplinary contexts.

**Clark Wells** Grand Valley State University

### Where Did That Come From?

A common experience of students in mathematics is what I call the "Where did that come from?" moment. This is the feeling that, while they follow the logic and understand the argument or process, they would never have thought of it themselves. From definitions to algorithms to theorems, mathematics is often presented to students in a way that obscures the process of doing mathematics. I contend that this is largely a result of the way we write, and teach students to write, mathematical arguments, calculations, and processes as purely deductive, consisting of pre-determined steps. By decoupling exploration from verification, we risk communicating that the steps we follow are things that, somehow, we are all supposed to "just know." Even inquiry based learning can fall victim to this problem. In this talk I suggest an alternate approach, where we encourage students to learn the landscape of problems, and to draw a detailed map, as it were, rather than follow the turn-by-turn directions of some mathematical "GPS navigation." I will present activities that allow students to not only discover isolated results, but tie them together into a cohesive net of concepts.

**Brian P. Katz** Augustana College

### Supporting Students' Defining as a Mathematical Practice

Defining is a key move in mathematical practice, but mathematical writing often presents definitions fully-formed, timeless, and without human agency. Asking students to participate in defining through mathematical inquiry therefore requires additional pedagogical moves. In this talk, I will compare my efforts to support students' defining in contexts that range from lower-division courses with definitions in a textbook to upper-division open-ended inquiry. Giving students agency over the definitions in our classrooms is vital for rehumanizing mathematics because it supports their authority and membership in our disciplinary and classroom community.

## Part C: Saturday, August 4, 9:00 AM–12:15 PM, Governor's Square 14, Plaza Building

**Na Yu** Lawrence Technological University

### Improving student learning in multivariable calculus through research projects

At the College of Arts and Sciences of Lawrence Technological University we implement a large-scale program of Course-based Research Experience (CRE), in which students practice research projects as part of a regular course. Multivariate calculus is part of CRE program. In this paper we will share what research projects were incorporated, how the projects were implemented in the regular course, how students responded and performed in the class and the benefits of the undergraduate research experience intervention.

**Joe Benson** Macalester College

### Calculus 1 + IBL + January term =

Teaching Calculus during a 4 week January term is a daunting prospect. There is a lot of material to cover and absorb in a short time. Taking an IBL approach to Calculus can be an effective way for students to properly engage with the while material building confidence, strengthening communication, and learning the content. This talk will discuss my experience teaching Calculus 1 using IBL during a January term – what worked and what could be improved in the future.

**Ann L. Von Mehren** Bowling Green State University

#### Encouraging STEM Student Self-Consciousness about "English-Units"

In a graduate data-analysis course, I conducted a focus group in which students were asked to discuss package information comparing weights, showing examples of bottles and wrappers where both English and metric units were plain to see. I wanted to know how many students would notice that some gave English units first, some gave metric units first. The group was approximately half foreign students and half American students. I was shocked at the vociferous reaction from the foreign graduate students, including students from Poland, Denmark and China. They immediately expressed anger at their daily need to convert English units to what is to them the more scientific metric system, at grocery stores, shops, restaurants and gas stations. They sarcastically illustrated being asked if they wanted a "foot-long" sub. They described reacting to mileage signs and getting the distance wrong when driving. All the American graduate students in the class said they had never thought about converting between units. When they had traveled or studied overseas, they said they silently made hand gestures at stores or filled their car tanks at the gas station, without bothering to convert from metric to English units. My paper argues that American students need to be challenged to inquire and learn how to overcome what they take for granted about the metric system of measurement. I am not seeking to advocate for the U.S. to convert to metric, but rather to encourage teachers to find ways to get American students to accept the challenge of thinking about the difficulties involved in mathematical measurement if they are not fluent in the international scientific measurement system.

**Benjamin Gaines** Iona College

#### Overcoming Resistance to Inquiry-Based Learning in a Math for Liberal Arts Class

Inquiry-Based approaches are a natural fit when teaching a survey math course designed for non-majors. The topics are typically new to many of the students, which encourages them to reach their own conclusions and develop the critical thinking skills that are often the focus of such a course. In addition, since there is often not a specific content endpoint that must be reached, there are no concerns about coverage. However, some students are so used to the methods of teaching they saw in high school that they resist working on material without firm instructions on what they should be doing. We will discuss the topics covered in our Mathematical Thinking course, and the methods that we used to acclimate students to an inquiry-based approach. In particular, we will focus on how the particular topics covered can facilitate these methods, how "mini-lectures" at the beginning of each new topic can prepare students to do their own problem-solving, and how scaffolding through a good choice of examples can build student confidence and help them reach more general conclusions. We also will include student feedback about the method of instruction, and how we plan to use it to continue to modify the course and materials in the future.

**Martha Byrne** Sonoma State University

#### Assessment Projects in an IBL Proofs Course

In an Inquiry Based Learning course, standard examinations stand out as relics of a different system that emphasizes performance over mastery and are not as rich as the learning experiences of the course. This presenter has developed several projects and a final portfolio for an intro to proofs class that allow students to demonstrate growth and mastery while providing rich learning opportunities and a venue for reflection. She will present on one project and the final portfolio for this class including discussion of student outcomes and reflections.

**David Clark** Grand Valley State University

#### Supporting Unconfident Proof Writers in IBL Euclidean Geometry

Euclidean Geometry is a required course for all pre-service math teachers (and many math teaching minors) at Grand Valley State University. These students typically take geometry shortly after their first university-level introduction to mathematical proof. They are often intimidated by the process of creating and writing proofs, are skeptical of the purpose of proof, and lack confidence in their own mathematical abilities. We will describe and reflect on an IBL approach to this class, with the help of student reflections and survey responses. This IBL class has the goal of introducing students to the real process of mathematical conjecture and proof while providing many supports, including the use of mastery exams, readings focused on productive failure, and assessments designed to encourage reflection and revision.

**Paul E. Becker** Penn State Behrend
**Mark Medwid** Rhode Island College

#### Matrix representations as a gateway to group theory

Computer algebra systems are widely available in college classrooms. In this environment, matrix representations of groups provide a unifying concept across abstract algebra, linear algebra, and geometry. Almost all finite groups encountered by undergraduates admit representations as multiplicative groups of concise, block-diagonal, binary matrices. Such representations provide simple, visual examples for an introductory group theory course. More importantly, students can manipulate these models to explore more abstract concepts. We describe computer labs that explore homomorphisms, isomorphisms, normal subgroups, cosets, and quotient groups. These labs lead to several standard theorems of abstract algebra.

**Ian Whitehead** Macalester College

### Teaching Number Theory From Scratch

I will describe an elementary number theory course built around a single problem, in which students were challenged to gather data, formulate a conjecture, and prove it as a team. The course was an honors seminar at the University of Minnesota, pitched to students with a strong interest in math but not many college courses. Students got acquainted with the rewards and challenges mathematicians face in generating original work. I'll discuss how I structured the course, my main takeaways from the experience, and how it might adapt to other contexts.

**Christopher ONeill** San Diego State University
**Lily Silverstein** University of California Davis

### Discovery learning in an interdisciplinary course on finite fields and applications

Math 148 at the University of California Davis covers finite fields and some combinatorial applications such as block designs and error-correcting codes. Teaching this course presents some unique challenges, as the algebraic content is on the advanced end of the upper division mathematics curriculum, but the students, which are typically half math majors and half computer science majors, have highly varied backgrounds (e.g. some have not yet seen modular arithmetic at the start of the course). In this talk, we discuss two sections of Math 148 taught using a partially flipped classroom, with two lecture days and two discussion-based "discovery learning" days each week. The in-class group activities focus on conveying intuition for more abstract concepts without diving too deep into technical machinery, and provide a natural venue for glimpsing related topics like projective geometry that are not typically seen in upper division computer science courses (or even upper division mathematics courses).

**Jonathan Weisbrod** Rowan College at Burlington County

### A Technique to Discover the Cauchy-Euler Equation While Reviewing Differential Calculus

I am sure I am not the only one who has reviewed differential and integral calculus concepts in an ordinary differential equations course. It is always a good practice to encourage questions from our students but what if the students choose not to ask, possibly due to fear of embarrassment? The problem presented in this talk is designed for students to review applications of differentiation and transformations of functions in order to answer the questions correctly. In class, this activity encourages review and mastery of topics in differential calculus in order to complete it. The outcome is the discovery of the Cauchy-Euler Differential Equation and its relationship to solutions of the form $y=x^m$.

# Mastery Grading

## Part A: Thursday, August 2, 1:30–4:05 PM, Governor's Square 11, Plaza Building

**Organizers: David Clark** Grand Valley State University
**Robert Campbell** College of Saint Benedict and Saint John's University
**Jeb Collins** University of Mary Washington
**Alyssa Hoofnagle** Wittenberg University
**Mike Janssen** Dordt College
**Austin Mohr** Nebraska Wesleyan University
**Jessica OShaughnessy** Shenandoah University
**Cassie Williams** James Madison University

"Mastery grading" refers to a suite of assessment techniques that encourage students to pursue deep understanding of course content. Techniques include standards-based grading, specifications grading, and mastery testing. Grades are based on mastery of objectives rather than accumulation of partial credit. Students have multiple attempts to attain this high standard for each objective, teaching them to persevere through the course.

**Kate Owens** College of Charleston

### A Quick Summary of Four Years of Standards-Based Grading

Over the last several years, I've implemented standards-based grading in courses ranging from college algebra to vector calculus. In this talk, I'll share with you what has gone well, what modifications I've made along the way, and what changes I'm considering for upcoming semesters. Additionally, I'll offer some practical advice for anyone interested in moving toward a standards-based grading system.

**Michelle A. Morgan** University of Northern Colorado

### The Unstandardized Nature of Standards-Based Grading Practices in Middle School Mathematics Classrooms

This qualitative, multicase study sought to describe middle school mathematics teachers' teaching practices as they implemented stands-based grading. Specifically, the study focused on better understanding the teachers' implementation of standards-based grading, use of assessment and feedback, and instructional design. Guided by a cultural-historical activity theory lens, the researcher invited six middle school mathematics teachers who self-reported using standards-based grading practices to participate in the research study. Data collection for each case consisted of two interviews, lesson summaries and reflections, and classroom observations over the course of four to five consecutive class periods. Using a combination of cultural-historical activity theory and discourse analysis, the researcher analyzed the data for themes both within and across cases. Preliminary data analysis suggests significant differences in interpretation and implementation of standards-based grading practices and policies within and across school districts. The presenter will offer recommendations for policy, implementation, and instruction.

**Chris Lee** Roanoke College

### Mastery Based Testing - A Case Study in Implementation Across a Mathematics Curriculum

Over the last two years at Roanoke College I have implemented Mastery Based Testing in a wide variety of courses across our Mathematics curriculum including: general education Introductory Statistics, Calculus & Multi-Variable Calculus, Linear Algebra, and Operations Research. Painstaking data has been collected on more than 2800 mastery attempts by students. In this talk I will present and summarize findings regarding mastery rate, type of topic, timing and frequency of testing opportunities, and other qualitative results such as student motivation and perceived difficulty measured from surveys. Specific attention will be given to drawing comparisons of effectiveness and perceptions between different cohorts of students in different levels and types of courses.

**Jane Zimmerman** Michigan State University

### Implementing Standards-Based Grading in a Post-Secondary Mathematics Course

Standards-based grading is an assessment/grading system that attaches scores to the mastery level of specific learning objectives. This methodology is gaining momentum in the K-12 environment but has been largely overlooked at the post-secondary level. In this paper, we will discuss the implementation of this practice in a College Algebra course at Michigan State University. We will examine the motivation to investigate an alternative assessment system, review the challenges faced in the development and implementation process, and describe the resulting system. Comparisons will be made between a College Algebra course utilizing the traditional points-based system and the same course in which standards-based grading was incorporated. These comparisons will include observed student behaviors, student perception, student performance in subsequent courses and the administrative efforts required of the instructor. Although the pilot courses were small classes, most College Algebra classes at MSU are large lectures. The feasibility of scaling this system to meet the needs of the general College Algebra population will also be addressed.

**Amanda Harsy** Lewis University

### Does Mastery-based Testing Help with Test Anxiety? Growth Mindset? Confidence? An Analysis of the Impact of MBT in Mathematics Courses

As educators, it is important to recognize that our assessment methods affect student attitudes. If we want students to learn from their mistakes and counteract a fixed-mindset of learning, perhaps we should look at what we incentivize in the classroom. One way that professors are attempting to encourage a growth-mindset of learning is through using and researching a new assessment model called "Mastery-based Testing" (MBT). In this talk, we present the results of research conducted in MBT courses over the past few years. Some of the questions we explored included whether the students' attitudes about mathematics changed throughout the course, whether there was a difference in attitudes and perceptions between calculus students and applied linear algebra students, whether MBT seemed to impact test anxiety, and whether or not there was a difference in attitudes and perceptions between MBT students and students who had traditional assessment.

**Jessie K. Lenarz** St. Catherine University
**Kristine Pelatt** St. Catherine University

### Combating Test Anxiety in Under-represented Groups

Recent mathematics education research has provided data supporting the claim that student engagement is correlated with persistence through STEM courses. The effect is greater for groups historically under-represented in mathematics, such as women and students with test anxiety. Mastery-based exams provided one tool to combat test anxiety in mathematics and increase a student's ownership of her own learning. We discuss our implementation of mastery-based testing in courses with active based learning as well as a traditional lecture course. In particular, the logistics of organizing a course for sufficient student feedback and meeting the needs of students with accommodations will be addressed. We also discuss the successes and pitfalls we encountered and propose solutions to some of these issues.

**Thomas Mahoney** Emporia State University

#### Communicating Student Progress in Standards-Based Grading

One of the hurdles to implementing mastery-based grading systems is the lack of tools available to clearly communicate students' progress in the course. I incorporated standards-based grading in College Algebra, Linear Algebra, and Calculus II during the 2017-2018 academic year. When setting out with my standards-based experiments, my goal was to avoid student confusion about computing grades at the end of the semester. I will share the methods of setting up grading systems, tracking students' multiple attempts at each standard, and how to efficiently share their current progress using custom live webpages and emailed progress reports.

**Anil Venkatesh** Ferris State University

#### Mastery Grading for the Masses: A Public Reflection

Interest in mastery grading has blossomed in recent years, with a recent MathFest minicourse, contributed paper sessions, and forthcoming special edition of PRIMUS. These venues have helped practitioners loop their innovative practices back into the community, contributing to substantial ongoing adoption of mastery grading principles. Yet many colleagues who have experimented with mastery grading have met with resistance or outright hostility from their departments. In this talk, we propose a set of best practices for engaging with non-practitioners on the validity and merits of mastery grading. We also aim to start an audience discussion on future directions for the mastery grading community.

## Part B: Friday, August 3, 1:30–5:05 PM, Governor's Square 11, Plaza Building

**Jason Elsinger** Florida Southern College
**Drew Lewis** University of South Alabama

#### Standards Based Grading adopted to Mathematics Courses

Standards based grading is an innovative grading scheme which is based on a student's mastery of course learning outcomes or standards. Final grades are determined by the instructor's choice of specifications for each letter grade. One key feature is that students can initiate re-assessment opportunities which can replace earlier scores. In this talk, we will describe the standards based grading scheme and how it was adopted to some mathematics courses. We will also describe the benefits observed, and efficient ways to implement the scheme including keeping a grade book and how to set up office hour re-assessments.

**Justin Wright** Plymouth State University

#### Mastery-Based Testing in Calculus: The Easiest Hard Test Questions

We discuss some of the question types that can be used when using master-based testing in a Calculus 2 and 3. When using partial credit grading, instructors may feel limited in the formats and types of questions that can appear on an exam as the awarding of partial credit must be considered. These constraints may be removed when an instructor employs mastery-based testing allowing for questions that emphasize conceptual understanding rather than computation and manipulation. Example questions will be presented along with a discussion of the benefits of the conceptual questions. These questions are often rather easy to answer but appear to be difficult to students.

**Sharona Krinsky** California State University Los Angeles

#### Mastery Based Grading in the Calculus Classroom: Increasing Rigor, Improving Transparency, and Empowering Student Success

In the current educational environment, professors face increasing demands for student-centric learning, measurable teaching and learning outcomes, and higher customer "satisfaction. Find out how mathematics professor Sharona Krinsky uses mastery grading in her Calculus classroom to maintain and increase rigor while meeting these increasing demands. By focusing on mastery of specific content and process standards, Professor Krinsky is able to more accurately gauge student learning, empower students to fail forward, and increase student confidence and preparation for further mathematics. Through low-stakes quizzes and rubric based grading students understand where they stand and have clarity on what they need to do next to accomplish their goals. In this session, Professor Krinsky will discuss the standards she uses in the Calculus classroom, demonstrate the use of re-assessment quizzes to determine mastery, as well as discuss the use of mathematical process standards to increase the level of task complexity and accuracy that students are expected to develop.

**Sharon Lanaghan** California State University, Dominguez Hills
**Kristen Stagg** California State University, Dominguez Hills

#### Redesigning Calculus I with Standards Based Grading and Active Learning with Technology

Despite the importance of Calculus I in STEM students' path toward graduation, pass rates in the course have typically been low, with only 60.9% of students earning a C or better in the 2015-2016 academic year. Many students lack both content prerequisite skills and 21st century learning skills (critical thinking, creative thinking, communicating, and collaborating) required for success in the course and in their future STEM courses. Our redesign uses standards based grading, active learning with technology and the use of Supplemental Instruction Leaders (SIs) to help students identify and remediate skill gaps, offer students multiple ways to interact with content, and give them opportunities to develop communication and collaboration skills. Initial results from Fall 2017 suggest that these changes have had a positive impact on student success. In this session, we will describe our redesigned course and its outcomes, including the impact on the STEM Mathematics Pathway at CSUDH.

**John Ross** Southwestern University

#### Implementing Mastery-based Quizzes and Tests in a Calculus Course

We describe several mastery-based approaches to Calculus 1 and Calculus 3 that have been tested and improved on over the last few years. We use two major types of assessments - quizzes and tests - that are graded in slightly different manners. We discuss the reasoning and the evolution of our approaches, as well as practical considerations that come with so much assessment.

**Austin Mohr** Nebraska Wesleyan University

#### Using Mastery-Graded Homework to Promote Perseverance

In a mastery-based grading scheme, one sets criteria for acceptable and unacceptable work as opposed to issuing a numerical score corresponding to partial credit. By setting a clear, discrete level at which homework credit is obtained, students are required to persevere through challenging work in instances where they might have otherwise accepted partial credit. The safety net of partial credit is replaced with the opportunity for revision of unacceptable work, allowing students to respond to instructor feedback where it is most useful. I will outline the mastery-based framework I've used for homework assignments in second- and third-semester Calculus and offer reflections on its effectiveness.

**Emma Wright** Plymouth State University

#### On Mastery Grading in Proofs-Based Classes

How do we grade a proof? How do we grade a computation? How can we equitably, concisely, and quickly grade both types of exercises? In this talk, we discuss a mastery-based grading scheme applied to weekly homework exercises in Geomeries and Abstract Algebra. This system encourages students to learn from their mistakes by allowing students to resubmit their work. This talk will discuss the impact of mastery grading, the structures in place to help guide students to mastery, and the time commitment for the instructor. Example rubrics and guidelines will be provided.

**Bevin Maultsby** North Carolina State University

#### It's Binary: Using Mastery Grading to Motivate Students to Become Good Coders

Following Mathfest 2017, we implemented mastery grading in a Matlab programming course for math and engineering majors at NC State University. Students are motivated to master a list of 16 tested programming techniques which encourage good code development. Under traditional grading, students who began the course with low computer proficiency were penalized for early poor performances. In the current version, students are assessed primarily based on what they know by the end of the semester, which motivates students to focus on areas where they can improve. We will discuss some shortcomings from Fall 2017 and how they were improved in Spring 2018.

**Chad Wiley** Emporia State University

#### Using Specifications Grading to Improve Students' Proof Writing Skills

In this talk I will discuss how I have implemented some of the ideas of specifications grading into the graduate courses I teach at Emporia State, particularly our introductory course Mathematical Proofs. The primary focus will be on how I use the ideas of pass/fail grading and student revisions to encourage students to think critically about rigor, clarity, and brevity in their proof writing. I will also describe how this grading method differs from methods I used in the past and how successful I believe the change has been.

**Jacob Price** University of Puget Sound

#### Mastery-based grading in a senior level probability and statistics course

I implemented mastery-based grading in a senior level probability and statistics course geared towards students interested in pursuing computational finance graduate degrees. Mastery-based grading assessment was incorporated into every evaluation instrument of the course. The all-or-nothing nature of mastery-based grading allows one to require students to use proven methods for improving learning, and one hopes this will carry over into other courses. Student performance and impressions will be presented. I will provide commentary upon student experiences, my own impressions and reflections, and the pros and cons of my chosen implementation.

**Sarah J. Greenwald** Appalachian State University
**Amy Ksir** United States Naval Academy
**Megan E. Selbach-Allen** Stanford University
**Jill Thomley** Appalachian State University

#### Raising the Bar with Standards Based Grading

The four of us have used Standards Based Grading in four very different contexts: calculus, statistics, introduction to proofs, and a first year seminar course. For the past year we have engaged in rich conversations about the commonalities and differences we have observed. In this talk, we will provide an overview of our four implementations, and share what we have learned from our conversations. Through our practice and conversations we have found that standards based grading makes the connection between learning and grades more direct and transparent to students. It also allows us to ensure that students experience success in key portions of our courses. We like that it aligns well with developing a growth mindset in students. The largest challenges we have faced relate to time management, both for ourselves and our students. We will discuss the positive impacts on classroom culture that we have all noticed, and describe the common challenges we have faced, and the different ways in which we have addressed them. We hope attendees will leave this talk with ideas to support their own implementation of SBG or similar grading schemes.

## Part C: Saturday, August 4, 1:30–3:05 PM, Governor's Square 11, Plaza Building

**Drew Lewis** University of South Alabama

#### Factors Affecting Student Participation in Voluntary Reassessments in SBG

One key feature of an effective Standards-Based Grading (SBG) scheme is the opportunity for students to initiate reassessment attempts. Over the last few years, we have noticed different types of students taking advantage of these opportunities at different rates; for example, the median number of office hour attempts by women is consistently higher than the median for male students. In this talk, we report on the results of a study on how factors such as mindset, anxiety, and susceptibility to stereotype threat are related to the number of office hour reassessments attempted by students in an SBG Linear Algebra course.

**Steven Clontz** University of South Alabama

#### Tools to Facilitate Mastery Grading

One of the barriers to mastery grading is the lack of first-class support on major Learning Management System platforms. This presentation will review a suite of custom and commercial software/SaaS solutions to streamline the process of mastery grading. Features of this suite include storage of student data, analysis of student data, generation of progress reports for each student, and generation of custom assessments for each student based upon individual progress.

**Joshua Bowman** Pepperdine University

#### Alternative Assessment Methods: Five Years In

Since using standards-based grading for the first time in spring 2013, I have implemented a variety of "non-traditional" assessment methods for undergraduate classes ranging from calculus to real analysis. I will address how different assessment methods may be better suited to different types or levels of classes, how to face challenges like the time commitment and possible skepticism from colleagues, and how students' attitudes have been positively affected, leading to improved learning outcomes.

**Rebecca E. Gasper** Creighton University

#### Do students get it? SBG implementation at three levels of the curriculum

Specifications Based Grading was partially implemented at a medium-sized liberal arts college in three classes: Calculus I, Calculus II, and Introduction to Proofs. Despite best practices, complete explanations, contracts, syllabi, and reminders, students find new things to be confused about and new complaints; even when instructors focus on student buy-in, a lot can go wrong. This talk, based on student evaluations, informal evaluations, and discussions, recaps real student misgivings about the credibility and clarity of nontraditional grading systems at the introductory/core level, 200-level, and 300-level.

**John Prather** Ohio University

**Preparing Introductory Math Students For What Comes Next: Using High Stakes Quizzes Early (And Often)**

Throughout my career, with my introductory math students I have used what I affectionately call "special" quizzes. While I use different variations in different classes, most of the time, this is a relatively short, single quiz over material that is basic to the class, and that I need students to understand well before they can proceed to more difficult concepts. They are high stakes in the sense that students must get a perfect score by a certain date, or else fail the course. They can take the quizzes as often as needed though, and I work with the students as much as necessary to achieve these perfect scores. In this talk, I will discuss the variations of these quizzes as they apply to the different classes I teach. There will be special emphasis on using them to prepare developmental mathematics students at the beginning of the course. Considering both the advantages and some disadvantages of these quizzes, the net effect on the courses has clearly been positive. After all of the students have mastered the essential (usually pre-requisite) material, the remainder of the course becomes much smoother both for them and for me.

# Mathematical Themes in a First-Year Seminar

## Thursday, August 2, 1:30–5:05 PM, Governor's Square 16, Plaza Building

**Organizers: Jennifer Schaefer** Dickinson College
**Jennifer Bowen** College of Wooster
**Mark Kozek** Whittier College
**Pamela Pierce** College of Wooster

As mathematicians, we are eager to infuse our discipline into First-Year Seminars, which often serve as an introduction to college-level academic culture (critical reading, writing and thinking, information literacy, etc.). Speakers will share their seminar's topic, major learning goals, the ways in which mathematical themes were incorporated into the seminar, and the degree to which their pedagogical choices were successful.

**Matthew J. Prudente** Saint Vincent College

**Seminar Precalculus Through Applications**

A first year seminar class at a small, liberal arts institution is a great way for students to build a sense of camaraderie with their fellow classmates. At Saint Vincent College, our goal with the first year seminar program is to focus on areas essential for success: academic responsibility, critical thinking and presentation of ideas. As a first-year seminar instructor in a precalculus class, I deliver the material in such a way that reflects how the mathematics they learn in precalculus will influence their lives through practical examples and small group projects.

**Gretchen W. Whipple** Warren Wilson College

**Math Anxiety Investigated as a FYS**

All first-year students at Warren Wilson College take a First-Year Seminar. This course has the modest goals of introducing a discipline or an interdisciplinary topic, enhancing critical thinking skills, nurturing a sense of civic identity, improving written and oral communication and argumentation skills, and developing a sense of personal identity and sense of place. Several mandatory writing assignments as well as community service experiences function as vehicles for some of these goals. I have taught a variety of math themed First-Year Seminars since 2004. It has been challenging to address some, never mind all, of these goals. Investigating math anxiety allowed us to address most of the course goals without forcing the instructor to teach material outside of her field of expertise. This talk will address the structure of this course, what was and was not effective, the student response, and why every math professor should consider teaching the topic.

**Amanda I. Beecher** Ramapo College of New Jersey

**Measuring Sustainability**

The First-Year Seminar course "Measuring Sustainability" was developed to be writing intensive, have a theme consistent with the College's mission, and help students transition from high school to college life both inside and outside the classroom. The course explored quantitative evaluation models within the topic of sustainability to understand the effects of our personal decisions and (College) community decisions on each other as well as the Earth. The culminating experience was a campus sustainability project, in which groups of 3 or 4 students analyzed and proposed a solution for a campus partner to improve current practices towards the sustainability goals of the College's strategic plan. Every team used a quantitative component in their evaluation or solution method, despite it not being a formal requirement of the project.

**Kathryn Cerrone** The University of Akron

### Experiential Learning & Statistics in a First-Year Seminar Course

Learning communities can be a wonderful way of bringing students together across multiple courses, to build relationships and foster learning. After several years of teaching the first-year seminar component of a statistics learning community, my teaching team decided to incorporate experiential, service learning in order to engage the students on a deeper level. We chose the topic of immigration not only because it is a trending topic of discussion but because it is one that many people have little first-hand experience with. We gathered data on the local population, partnered with an immigration support organization and integrated this topic into several assignments throughout the semester. We used time in and out of class to discuss how math and statistics can provide a model of a situation but sometimes personal interaction is needed to get the full picture with color and context. During this talk I will present details on this project as well as feedback from the students on their experiences. I will also address some pitfalls that were encountered and how they were addressed in subsequent years.

**Cynthia Farthing** University of Iowa

### Uncovering the Hidden Figures

This presentation will focus on a first-year seminar course, Uncovering the Hidden Figures, developed at the University of Iowa based on the book Hidden Figures by Margot Lee Shetterly. The book tells the story of the human "computers" employed at NASA and their contributions to space exploration while shining a light on racial and gender discrimination the women endured. This book was a starting point for several course topics: the participation and achievements of women in STEM field from WWII to today, the role of women in the early development of computers, how technology has changed the way science is done, and how we can increase the number of underrepresented groups in STEM fields. Much of the course was based on discussion of the book and other related readings. The learning objectives for the course included the ability to: describe how computers and other technology influenced mathematics and science, identify factors contributing to increased participation by women in STEM fields, identify factors that may prevent women from participating in STEM careers or factors that may cause women to leave STEM careers, and investigate and describe a current area of research in science and technology. Topics from the course and sample activities will be shared with the audience. The most popular activities involved exploring the effects gender schema and stereotypes that may influence decisions to enter STEM fields, discussion with current STEM faculty about their careers and achievements, and experimenting with slide rules. We will also discuss adjustments to the course that will be made in the future and opportunities to make a course like this more interdisciplinary (for example, by partnering with humanities or social science faculty).

**Kim Spayd** Gettysburg College

### Math and Art in a First-Year Seminar

In the Fall 2016 semester, I taught a first-year seminar entitled "Math as Muse: Exploring the Relationship Between Math and Art" at Gettysburg College. The learning goals were, first, to gain an appreciation for how artists have incorporated various geometric elements into their work, and second, to gain a targeted introduction to academic rigor and expectations at the collegiate level. Class sessions were primarily devoted to discussions of how certain geometric elements contributed to particular works of art or to the creation of an artwork around a central theme. The course was structured according to spatial dimensions with many topics and activities within each unit. Students measured ratios in portraits to gain insight into artists' intentions and viewers' potential responses. We discussed the symbolism of different polygons in abstract modern art and created works using only simple two-dimensional shapes. The visual impacts of symmetry were highlighted by watching choreographed dance sequences from musicals. Platonic solids and exotic three-dimensional models were featured prominently in the context of abstracted human forms. Students read and discussed *Flatland*, then considered artwork from Cubist and Surrealist movements through this fourth-dimensional lens. Two large-scale papers during the semester were used to assess comprehension of the mathematical topics and their roles in art. These papers, along with smaller assignments and weekly co-curricular group meetings, were used to further the learning goals associated with acclimating to a college environment. Students ended the semester with a deeper appreciation for the role of mathematics in art and further along the learning curve for collegiate success.

**Jennifer Schaefer** Dickinson College

### Mathematical Identities: Diverging from the Stereotypes

In this talk, we reflect on a first-year seminar designed to analyze how mathematicians are portrayed in popular culture and how these portrayals involving gender, sexual orientation, race, and mental health affect who is attracted to, and often welcomed into, the field of mathematics. We will discuss the structure of this writing-intensive, discussion-based course and provide an outline of course topics, readings, films, and writing assignments. David Hilbert has been quoted as saying, "Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country." Through this study of portrayals of mathematicians in popular culture and the implications of these portrayals, did our students discover mathematicians have one, distinct identity as well?

**Sarah Mayes-Tang** University of Toronto

**A first-year seminar on creativity in mathematics**

The "Creativity Theme Stream" was a sequence of four first-year seminars about creativity in math, science, social science, and humanities. In this talk I will describe my experiences teaching the mathematics part of the sequence, and how I modified an existing first-year seminar on modern mathematics to focus on the creativity theme. I will also share three ideas for integrating creativity into any first-year seminar.

**Allegra B. Reiber** University of Denver

**Mathematics Through Fiction: Creatively Exploring Mathematical Thinking and the Nature of Mathematics**

The goal of my First Year Seminar, "Mathematics Through Fiction," is for students to explore mathematical fiction as a way to teach and understand mathematics, its nature, and its community. Through reading and writing about mathematical fiction, exploring mathematical problem solving, and creating fiction that teaches mathematics, each student will develop their own understanding of mathematics, creativity, and their own mathematical voice. We reflect on how one can learn mathematics differently and how math comes to life through fiction. The course readings, *The Number Devil*, *Flatland*, and short stories exploring number theory, infinity, incompleteness, topology and the fourth dimension, introduce the mathematical topics that we study through group activities and mathematics exercises: creating divisibility rules in different bases, devising algorithms to accommodate countably infinite guests at Hilbert's Hotel, and observing cross sections of three-dimensional solids passing through the plane. Critical discussions and writing about the readings have students examine how mathematics and mathematicians are represented in literature and whether the messages in the pieces promote productive perspectives on mathematics, who it is for, and why it matters. Students create two pieces of their own mathematical fiction, joining the tradition they study. Critical thinking and mathematical problem solving skills are explicitly developed using the framework and language of *The 5 Elements of Effective Thinking*. The final exam includes mathematics problems and a capstone portfolio, where each student documents their learning and thinking in the multiple realms of the course.

**Mary Shepherd** Northwest Missouri State University

**Exploring Mathematics Related Fields–A first-year seminar for mathematics students**

We designed a second semester first-year seminar for mathematics and mathematics education majors to give them experience in mathematics outside the teaching arena. The course is 1 credit hour and involves three projects and some job search skills. One project is designed as a pure mathematics experience, one as a data science experience and one as an applied mathematics experience. The course has now run three times in the spring semester. The learning goals are relatively simple, to experience, in as authentic a manner as possible each of these three types of mathematics. Students work in groups of four with the groups changing for each project. The output of each project is different, a journal-article type paper for the pure mathematics project, a powerpoint presentation for the data science related project and a presentation which could be a proposal of some type for the applied mathematics project. The success of the student is based on the quality of the experience the student has, not necessarily on the quality of the final result, as it is not always possible to get a desired result in the short timeframe that exists for the course. Students also gain experience in creating a resume, participating in mock interviews and attending a career fair.

**Emlee Nicholson** Millsaps College

**Cryptology in a First Year Seminar**

Seminar title: How Safe Are Your Secrets? Student Learning Outcomes (adaptations of definitions from the AAC&U rubrics): (1) Integrative and Collaborative Learning (2) Problem Solving & Creative Practice. Millsaps College launched a new general education curriculum including a skills based first year seminar. The seminar is discipline specific but developing students' problem solving and collaboration skills is the primary goal. In my course, students investigate the strengths and weaknesses of many known techniques in cryptology. Students study some of the history and mathematics of cryptography and cryptanalysis. Students work to decrypt messages with and without knowledge of the encryption technique or key. I teach math content on an as needed basis. Topics include set theory, functions and inverses, modular arithmetic, and operations with matrices including finding inverses. Assessments for projects focus on process. Students must communicate what they tried, how they knew it was or was not working, what they tried next and why, etc. I have taught this course three times. This talk will include both successes and failures of these iterations. Examples of Assignments and Activities: Collaborative: Students engage in a 4-week competition. Teams receive money to hide on campus and submit an encrypted message describing the precise location of their money. Teams intercept and attempt to decipher each other's encrypted messages for the next week. The team who deciphers the message first collects the hidden money. Individual: Students receive 3-5 encrypted messages of increasing difficulty. Some contain the encryption technique or other clues. Students attempt to decipher the messages.

# Mathematics and the Life Sciences: Initiatives, Programs, Curricula

## Saturday, August 4, 1:30–3:25 PM, Governor's Square 17, Plaza Building

**Organizers: Tim Comar** Benedictine University
**Raina Robeva** Sweet Briar College

The 2015 CUPM Curriculum Guide to Majors in the Mathematical Sciences identified the life sciences as a key path through the mathematics major to graduate programs and the workforce. Topics include scholarly contributions addressing initiatives, programs, curricula, and course materials at the interface of mathematics and the life sciences that have been implemented and tested at institutions of higher education.

**Margaret Rahmoeller** Roanoke College

### Quantitative Biology: An Alternative to Calculus for Biology Majors

While an understanding of several of the concepts in Calculus I is important for biology majors, many of the skills and techniques emphasized in the course do not seem directly applicable. Two years ago the biology department at Roanoke College changed their required math course for majors to a Quantitative Biology class, which was developed by myself and another math faculty member. We cover modeling without ever defining a derivative and statistics from a biological standpoint. We emphasize working with real data and use Minitab and Mathematica to help with the calculations. In this talk, I will share topics and concepts we cover and activities and case studies we use to encourage critical and creative thinking.

**Harry F. Hoke** University of Richmond
**Kathy W. Hoke** University of Richmond

### First-year Calculus Workshops using Biology Lab Data

We present an example of a workshop used in a first-year calculus course we teach to students who are also taking first year biology. A workshop is a weekly interactive group assignment completed in a single class period. The students, working in groups, use data collected in a biology lab to complete an analysis of that data within a framework we have designed to motivate and reinforce the concepts learned in introductory calculus. We have used this approach for the past five years in a yearlong Calculus I and II course. We highlight the positive effect on STEM retention achieved by using workshops to demonstrate the relevance of mathematics to the life sciences.

**Yanping Ma** Loyola Marymount University

### Reports on the attitudes of students in calculus of life science toward Mathematics in their careers

We developed an instrument to help identify the degree to which Calculus students' value mathematics skills in their prospective careers while modifying our curriculum to include more real-life applications. There are five aspects measured, including positive attitude towards the importance and relevance of mathematics/calculus towards their future career, as well their level of confidence, anxiety, and enjoyment about mathematics/calculus. This type of tool would support the quantitative tracking of attitudinal changes on students in Life Science over time and attitudinal comparisons across various subpopulations, including effects based on modifications of pedagogy. Factor analysis of the pilot population (Loyola Marymount University undergraduate students taking Calculus I for Life Science) will be done to test whether this instrument may accurately measure the five constructs. We will report results found through the instrument and share some projects we gave to students.

**Timothy D. Comar** Benedictine University

### A Pathway from Introductory Material to Undergraduate Research in Mathematical Biology

A two semester course sequence in biocalculus (calculus for the life sciences) is an great starting place for students in the life sciences to seriously begin applying mathematical and computational techniques to investigating biological problems and phenomena analytically. This talk discusses a variety of topics and activies in the course sequence that lead students to develop the skills and understanding needed to be able to use mathematics to address biological problems. With a broad, underlying theme of population dynamics, the activities take the students from investigating the relationships of simple dynamical systems to data sets and exploring simple models to studying biological mathematical models requiring several dimensions, reading journal articles, working through computer laboratory projects, and eventually leading to the detailed study of a sophisticated model in a published journal article. Students with this preparation subsequently have completed successful research projects in mathematical biology as well as majors and research projects in related scientific disciplines.

**Therese Shelton** Southwestern University
**Emma K. Groves** North Carolina State University

### Incorporating Biology Topics into Mathematics Undergraduate Experiences

We will share multiple cases of sprinkling biology into mathematics courses, including flu data to solidify the Fundamental Theorem of Calculus, the Monod equation in microbiology to model the force of infection in a modeling class, summer research on measles and cholera, and capstone projects on HIV and the generation and spread of the cancer cells of chronic myelogenous leukemia (CML). Some of the work was supported by institutional grants from the Howard Hughes Medical Institute and from the W. M. Keck Foundation.

**Kseniya Fuhrman** Milwaukee School of Engineering

### Mathematical Analysis of Oscillatory Network of Transcriptional Regulators as a Course Project

In this talk, I will present a model used as a course project in the undergraduate Mathematical Biology course. This model describes the behavior of an oscillatory network of transcriptional regulators. The mathematical analysis of this model combines analytical derivations as well as computer simulations. The students were asked to find the steady states of the system, perform stability analysis, and find the critical parameter values. In addition, they wrote a Matlab simulation for the model and confirmed their theoretical findings with the results of the simulation.

## Mathematics Research Experiences for K–12 Teachers and Students

### Thursday, August 2, 1:30–3:45 PM, Governor's Square 17, Plaza Building

**Organizers: Saad El-Zanati** Illinois State University
**Cynthia Langrall** Illinois State University

Presenters will share their experiences conducting mathematics research with teachers and students. Participants will be introduced to a variety of problems that are well suited for these research experiences. They will learn about the findings that have resulted from these research experiences as well as the influences on teachers' instructional practice and students' learning and dispositions toward mathematics.

**Saad El-Zanati** Illinois State University
**David Barker** Illinois State University
**Cynthia Langrall** Illinois State University

### Research Experiences for PreService and InService Secondary Mathematics Teachers: The Teacher-Scholar Concept

We report on two Illinois State University programs that are designed to engage mathematics undergraduates and high school teachers in research. The first program is a course that has been run at Illinois State every spring since 2004. The second program is a Research Experiences for Undergraduates (REU) Site for pre-service and in-service teachers. The REU Site is funded by the National Science Foundation and has been run every year since 2007. This presentation will describe the successes and challenges of these programs, sample research topics, components designed to help teachers translate their research experience to the classroom, and suggestions for implementation.

**Joel Jeffries** Iowa Sate
**Stephanie Zeppetello** East Leyden High School

### Translating the REU Experience to the High School Classroom: A Tale of Two Teachers

The REU site at Illinois State University is designed specifically for pre-service and in-service mathematics teachers. During our summer at the ISU REU we conducted research in Discrete Mathematics, developed the mathematical habits of mind required for research, and discussed how to translate these experiences to our future classrooms. The overarching goal of the program is to influence the mathematics education of our future students. In this presentation we will discuss how the REU experience has changed our teaching practice, provide examples of lessons that have incorporated elements of mathematics research, and offer advice to teachers who want to incorporate research-like experiences into their classrooms.

**Lindsey States** Miami University
**Kerry Hawken** Ball State University

### REU Math Camp: A Genuine Mathematics Research Experience for Urban High School Students

Each summer, on the campus of Illinois State University (ISU), a group of pre-service and in-service teachers anxiously await the arrival of a dozen high school students from the Chicago Public School (CPS) District. The CPS students are attending a Mathematics Research Camp and gaining a new perspective of the world of mathematics. During the weeklong camp, the CPS students will have the opportunity to explore unsolved questions in graph theory, make significant research progress, and present their findings. In addition, the Mathematics Research Camp is an important time for the participants in the Illinois State University REU for Pre-Service and In-Service Secondary Math Teachers (ISU REU). The Math Camp is developed and run entirely by the ISU REU participants. Prior to the Math Camp, the ISU REU participants were engaged in research in discrete mathematics, developed the mathematical habits of mind necessary for research, and discussed how to translate these experiences to the high school classroom. The Math Camp provided the REU participants an important opportunity to implement the ideas they have been working on all summer. In this session, we report on the CPS math camp, the experience of the CPS high school students, and what the REU participants learned from this experience.

**Anant Godbole** ETSU

### Research Conducted as Part of RET Supplements

I will describe a variety of results proved by members of my small 2-3 person RET programs over the years. Participants have taught at the elementary, middle, and high school levels, and the level of their research has varied accordingly. As time permits, I will present research results and projects on

- The "Every Seventh Child Starves" Theorem;
- An M&m theorem that mirrors the M&m conjecture;
- Universal Cycles of complementary classes;
- Standard planarizations of the complete graph;
- Omnimosaics in third grade;
- Variations on the Kaprekar theme; and
- The p q investigation.

**James Tanton** MAA

### Inspiring Mathematical Research via Twitter

Is it possible to inspire, and even conduct, mathematical research with math enthusiasts of all backgrounds and all ages from all around the globe in 280 characters or less? For the past eight years, Tanton has been tweeting a mathematical question a day (well, almost every day) and will share in this talk some of the results that have come of it.

**Jongryul Lim** Korea Science Academy of KAIST

### Some number theory research experience with gifted high school students

The theory of integer partitions is well-known and widely investigated by many number theorists. A partition of a given natural number n is a way of expressing n as a sum of natural numbers. It can be easily understood yet rich in its theory. We investigated a new type of partitions called alternating partitions which we came up with an idea from alternating series in calculus. We encountered some obstacles and was able to overcome by imposing some conditions. We obtained a genenral formula for alternating partitions and some alternating partition identities. In this talk, we want to share what we have experienced while conducting some number theory research with students in a gifted high school in mathematics and science.

**Jenna R. O'Dell** Bemidji State University
**Cynthia Langrall** Illinois State University

### Introducing Students in Grades 4-6 to Unsolved Problems

We will share our experiences engaging students in Grades 4-6 in problem solving using challenging, yet accessible, unsolved mathematics problems such as the Graceful Tree Conjecture and the Frobenius Coin Problem. We will report on a study conducted with young students attending an after-school program in the Midwest and will discuss how exploring parts of unsolved problems affected aspects of their mathematical dispositions.

# Modeling-Based Teaching and Learning in Differential Equations Courses

## Saturday, August 4, 1:00–4:55 PM, Governor's Square 15, Plaza Building

**Organizers: Brian Winkel** SIMIODE
**Lisa Driskell** Colorado Mesa University
**Audrey Malagon** Virginia Wesleyan University

This session features talks about modeling-based teaching in differential equations courses and descriptions of modeling activities in a course from speakers who are beginning to use modeling and those with more experience. Talks featuring real data (collected or cited) and a full modeling process for students are offered. Evidence of the success of individual approaches will be given.

**John T. Sieben** Texas Lutheran University
**Reza O. Abbasian** Texas Lutheran University

### Air Water Rocket as Class Project

In this class project we will develop a mathematical description of the air water rocket performance. Then we will film the deployment of such a rocket and analyze the actual performance using Video Physics. Finally we will return to the mathematical description and adjust parameters to bring the theoretical predictions more in line with the observations.

**Gerard Ornas** McNeese State University

### It's close to rocket science

To motivate science and engineering majors as to why they have to take Differential Equations, the author starts every semester by deriving a second order nonlinear differential equation to model a rocket's trajectory using only principles from freshman physics. This is done before even mentioning what a differential equation is. From here we tweak the model to show how this can work for nearly any discipline. Then refer back to it throughout the semester to illustrate the general themes throughout the course. In this talk, we discuss the derivation, tweaks, and semester long callbacks. Then discuss plans for bringing in modelling projects based upon this and carrying the ideas forward into other classes.

**Jim Fischer** Oregon Institute of Technology
**Tiernan Fogarty** Oregon Institute of Technology

### A Boundary Value Problem Modeling-Exercise: Beam Equation

This scenario is designed to lead students to discover and validate a differential equation that models the vertical deflection of a horizontal beam under different boundary conditions. In addition, students learn how the choice of units can affect computational accuracy when employing curve-fitting software (e.g. Excel or Matlab). For example, using centimeters versus meters can greatly affect the outcome. Vertical deflection occurs as a result of the weight of the beam alone, with no compressive force at the ends or distributed loads other than the mass of the beam. Data is collected to measure the vertical deflection from horizontal along the distance of the beam for several boundary condition situations. A mathematical model is arrived at for the vertical deflection and from this model the general form of the governing ODE can be inferred. Using one of the boundary conditions (such as embedded at both ends), students determine and use an approximated polynomial solution to estimate the parameters of the beam. Students validate the model by solving the BVP for one of the other boundary conditions.

**Marco Iglesias** University of Nottingham
**Zaid Sawlan** King Abdullah University of Science and Technology
**Marco Scavino** Universidad de la República
**Raúl Tempone** King Abdullah University of Science and Technology
**Christopher Wood** University of Nottingham

### Estimation of the thermal properties of a wall using temperature and heat flux measurements

In this talk, we consider an experimental case study conducted in an environmental chamber, with measurements recorded every minute from temperature probes and heat flux sensors placed on both sides of a solid brick wall over a five-day period [1]. We are interested in retrieving the thermal properties of the wall, i.e., the thermal resistance and the heat capacity per unit area. To this purpose, the unidimensional heat equation with unknown initial temperature and Dirichlet boundary conditions is used to model the heat transfer through the wall. We propose a model-based parameter estimation technique that reduces the bias error of the estimates of the wall parameters, compared to other approaches where the boundary conditions are assumed to be non-random. The error bias reduction is achieved marginalizing the boundary conditions that act as nuisance parameters. As a by-product of our approach, we show that adequate tools are available to recommend the users how to plan efficient experimental designs, for example, minimizing the duration of the measurement campaign. [1] Marco Iglesias, Zaid Sawlan, Marco Scavino, Raúl Tempone and Christopher Wood (2018). Bayesian inferences of the thermal properties of a wall using temperature and heat flux measurements. International Journal of Heat and Mass Transfer 116, 417-431.

**Glenn Ledder** University of Nebraska-Lincoln

### The Past, Present, and Future of Endangered Whale Populations

This case study focuses on qualitative analysis of a model consisting of a single differential equation for population of a prey species given a fixed population of predators. The behavior of the model, which can be determined using a nonstandard method for phase line analysis, depends in a critical way on the overall hunting capacity (number of predators times efficiency), a number that has gradually changed through technology and international agreements. The stable equilibrium population can change dramatically as this parameter moves through a bifurcation value, leading to sudden changes in population levels corresponding to small changes in hunting capacity. The various scenarios can be strung together to create a historical narrative of whale populations along with a prescription for restoring a sustainable hunting capacity.

**Ryan Miller** United States Military Academy

### Modeling with Differential Equations (MA153) Course Changes and Project Ideas

This paper describes the proposed changes to MA153, which is a first year modeling course for advanced math students at the United States Military Academy (USMA). The history of the course is discussed along with the challenges of emphasizing a model first approach. Students at the United States Military Academy are all required to enroll in a modeling course, a calculus course, and a probability and statistics course. MA153 is the advanced option USMA provides to students who enter the academy with prior calculus experience. Students are selected using a combination of placement exam scores, AP and or IB scores, SAT and ACT scores, an interview process with the program or course director, and a CEER score, which is USMA's attempt to quantify student academic potential. We discuss the transition of the course from "Mathematical Modeling and Introduction to Differential Equations," to "Modeling with Differential Equations." The proposed changes interweave the USMA mathematical modeling triangle of transform, solve, and interpret throughout the course. We also describe the final course project on SIR model applications to the spread of a rumor and the use of progress review sessions to guide students towards understanding and developing their own differential equation models.

**Jean Marie Linhart** Central Washington University
**Daniel Roelke** Texas A&M University

### Coexistence and Competition

Phytoplankton are photosynthetic organisms at the base of the food chain. Seemingly simple, they compete for nutrients, resulting in both competitive exclusion and in coexistence of species. Modeling phytoplankton growth and dynamics requires a system of ordinary differential equations for the populations and the nutrients. We will discuss how to get students started modeling this rich biological system and some of the many research questions that are being explored with this mathematical model.

**Peter G. LaRose** University of Michigan

### Modeling, Team Based Computer Lab Materials in Differential Equations: Implementation and Outcomes

We present a model for applied computer lab materials for a calculus II prerequisite differential equations course serving primarily engineering students. The labs are structured to emphasize active learning in pairs and teams of four, to emphasize the modeling of real world applications, and to develop and extend course material and students' understanding of it. We describe the materials developed and the context in which they are used, and present data suggesting that over and following a two year implementation period the materials' underlying goals are largely being met. We conclude with thoughts on further development and ongoing challenges.

**Namyong Lee** Minnesota State University, Mankato

### Discrete-Space Continuous-Time and Discrete-Time Continuous-Space Modeling

ODE (Ordinary Differential Equations) based modeling project/class is easier to introduce to undergraduate students. However, it often shows limitation, especially in tackling the real world phenomena or problems which have both time and space dynamics. Compare to that, in order to observe both space and time patterns, we need to introduce PDE (Partial Differential Equations) based modeling, which might be a big stretch for undergraduate modeling project/class based on ODE. In order to fill the gap, we introduce Discrete-Space Continuous-Time and Discrete-Time Continuous-Space Modeling with series of concrete examples from neurophysiology, population dynamics, and pattern formation that we had performed though our class.

**Christopher Brown** California Lutheran University

### Sequential Course Activities Constructing a One Predator Two Prey Model Incorporating an Allee Threshold and Indirect Prey-Prey Effects

At our institution, the Differential Equations course serves as one of the only mathematical modeling experiences, and so introduction and reinforcement of the modeling process is a primary course goal. To achieve this goal, we introduced a sequence of related projects in population modeling to parallel course topics and build on one another. This sequence culminates in the creation and analysis of a model for the golden eagle, island fox, and feral pig community in the Channel Islands. We will discuss this sequential project, student outcomes, and good practices for designing similar sequential projects.

**Hasala Senpathy K. Gallolu Kankanamalage** Roger Williams University

**Dynamics of gestational diabetes: A model-based analysis.**

In this paper we present the development, improvement and validation of a mathematical model for gestational diabetes. Gestational diabetes is a temporary elevated blood-sugar condition that occurs during pregnancy. However, this condition relies on many hidden factors including hormone dynamics during the pregnancy. A part of this project, we implemented a model-based teaching of differential equations in a couple of independent studies. As a part of the project, the model is validated by some existing clinical data. As such a model consist of many hidden variables and parameters, this project has certain open endings that can yet be improved in the future to develop more accurate model as well as to engage more students in an improvement and model based leaning.

**Leon Kaganovskiy** Touro College Brooklyn Campus

**Maxima Modeling for Differential Equations**

In this presentation I would like to share some Modeling tools which I developed for the Applied Differential Equations using freely available Maxima Computer Algebra System. Maxima is somewhat less powerful than commercially available Computer Algebra Systems such as Mathematica and Maple. However, it is free and perfectly adequate for the introductory level courses. I will present simulations for heat seeking missile model, slope fields, Lottka-Volterra and Competing Species models, etc...

# Priming the Calculus Pump: Fresh Approaches to Teaching First-Year Calculus

## Part A: Friday, August 3, 9:30 AM–12:25 PM, Governor's Square 16, Plaza Building

**Organizers: Chuck Garner** Rockdale Magnet School for Science and Technology
**Bob Sachs** George Mason University

Many first-year college calculus students have had a previous encounter with calculus in high school. These new college calculus students start calculus having seen much of the material, but with a weakness or a lack of confidence in some areas. This audience creates unique challenges to the instructor. This session seeks to share fresh approaches to engage this audience.

**Mel Henriksen** Wentworth Institute of Technology
**Emma Smith Zbarsky** Wentworth Institute of Technology
**Gary Simundza** Wentworth Institute of Technology

**Modeling with Calculus: the Practical and the Whimsical**

We have prototyped an approach to calculus that emphasizes conceptual understanding through guided rediscovery and modeling using both whimsical and real-world projects to engage students. Students begin with an exploration of functions and the relationships between displacement and velocity, through a study of buckling and oscillating pasta. Later, students move to projects involving more practical materials such as stone, steel and concrete, analyzing several civil engineering projects. The whimsical nature of analyzing the behavior of pasta provides a disarming entree to mathematical modeling in the beginning weeks of Calculus I. Later, in Calculus II, students apply the concept of accumulation first developed using an oscillating cantilevered spaghetti noodle to more practical problems: analyzing requirements for the construction of a curved dam, determining the volume of enclosed space in a proposed hotel to be built into an abandoned granite quarry or within a classical cathedral dome.

**Nathan Pennington** Creighton University

**Modeling the Physical World: An Integrated Math and Physics course**

Modeling the Physical World is a project-based integrated Mathematics and Physics course aimed at advanced incoming first-year students. The course is team-taught with a faculty member from the Physics department and the students receive credit for Calc II and General Physics I. This talk will address the structure of the course, finding the right students, and qualitatively discuss the outcomes of the course.

**Nela Lakos** The Ohio State University
**James M. Talamo** The Ohio State University

### Ximera and Calculus Coordination

Calculus at The Ohio State University is tightly coordinated. All students have a common course calendar and syllabus and common assessments (homework, quizzes, and exams). Ohio State is in the process of implementing a free, open resource textbook and homework platform that will save our students an aggregate 1.1M per academic year. Produced in Ximera, a program that converts Latex documents into interactive online content, the book and exercises have been customized by us for our courses. We discuss the significant benefits of creating this content and incorporating it into the coordinated environment.

**Alex M. McAllister** Centre College
**Joel Kilty** Centre College
**Alison Marr** Southwestern University

### Re-envisioning the Calculus Sequence

The mathematics faculty of Centre College and Southwestern University have engaged in a thoughtful re-envisioning of the calculus sequence that incorporates most of the standard topics of the calculus sequence only in a nonstandard order. Alternative themes for grouping and sequencing topics, and different approaches to scaffolding the complexity of topics have been identified with a particular eye toward such goals as incrementally increasing the challenge from one course to the next, incorporating more modeling and "real-life" applications, including "new" ideas in each course for students who have taken calculus in high school, and encouraging persistence. This talk presents two new versions of the calculus sequence, with particular attention to the first two courses in the sequence, and articulates the over-arching vision that guides each version. Among other things, this work is motivated by evidence that the standard approach "filters" traditionally underrepresented students, the extremely different levels of student preparedness, the diverse career and continuing education paths of students, the ubiquity of more sophisticated technologies, and the ready availability of large data sets that enable more realistic and more relevant applications. Ultimately, we aspire to encourage the persistence of all students through the calculus sequence and onto further studies in STEM disciplines.

**Joseph Spivey** Wofford College
**Matthew Cathey** Wofford College

### An Integrated Interactive Approach to the Calculus Sequence

Wofford College, a 4-year liberal arts college with around 1500 students, faced particular challenges in its first-year calculus sequence: the semesters are 13 weeks long, and each class meets for only 2.5 hours each week. Thus, only differential calculus could be covered in the first semester, leaving integral calculus for the second. Students taking only Calculus I weren't seeing the whole picture; students with AP credit for one semester had seen much of our second semester and missed the rigor of the epsilon-delta formulation of limits. So, we redesigned the two-semester curriculum, presenting derivatives and integrals side-by-side in the first semester, and postponing formalities to the second. We created a digital book (using Wolfram CDF) that is appropriate for our new curriculum. This medium allows interactive figures, which illuminate the concepts of change much more effectively than static figures. Also, the cost to students is significantly lower than using traditional course materials, since the text itself is available at no cost. The presenters will talk about the advantages, disadvantages, and challenges involved in this new approach to calculus, and will give a brief demonstration of the textbook.

**Sarah Hews** Hampshire College

### Calculus in Context: An Innovative Approach to Calculus

The Calculus in Context (CiC) sequence at Hampshire College is an innovative approach to teaching calculus that prioritizes the learning process. In these inquiry based courses, students learn how to engage in mathematics, take risks, and work in groups. By focusing on concepts, geometry, graphing, and numerical solutions before techniques, algebra, and closed form solutions, students who have have already taken a class or classes in calculus are challenged immediately, and students who enroll in the courses with no calculus experience and unpleasant experiences in algebra courses are able to engage with mathematics in a fresh way. Since students are discovering the concepts of calculus through group explorations, advanced students are able to engage deeper with the material and novice students learn how to fill gaps in their mathematical background. This approach to learning creates the environment so that when students learn limits, they learn about infinitesimals and the paradoxes of calculus and when students learn derivatives, they learn how and why quantities change using differential equations. For the last few weeks of the semester, the students work in groups on projects that could be novel research, applied problems, or historical explorations. This talk will describe the CiC courses, how they challenge students who have already taken calculus, and present techniques and topics that are transferable to calculus courses at other institutions.

**William T. Mahavier** Lamar University

### Calculus for Students who Already "Know" Calculus

What might a Calculus course offer the student already capable of computing basic derivatives, integrals and series, while still serving well the student entering with no calculus experience? Taking to heart the words of Jean-Jacques Rousseau in "On Education," we accept that "All the instruments of education have been tried save one, the only one precisely that can succeed: well-regulated freedom." A carefully crafted, student-centered, active-learning approach can be just the vehicle for well-regulated freedom. By using problems ranging from the standard rote problems to problems challenging the best of students we attempt to offer every student the maximal opportunity for growth. Well-regulated freedom requires that every student master certain material, while encouraging (pressuring?) talented students to explore more difficult material. Requiring independent learning and presentations assures development of the most coveted of industry skills: problem solving and communication. Another perk is that an apparently "lesser" student may sometimes have an insight that can be praised and respected by the best prepared students. With the increasing popularity of PreTeXT/MathBook XML we are migrating our open source IBL Calculus I,II & III text from a LaTeX/PDF document to a PreTeXT/HTML/Sage document which makes it more readily accessible via phones and tablets while still being easy to modify to fit individual instructors needs. These notes and their source are free to anyone wishing to use and modify and are available, in almost their current status at http://www.jiblm.org/mahavier/calculus/html/index.html.

**Rachel Grotheer** Goucher College

### Calculus in the Real World: Increasing Relevancy Through Data and Modeling

Goucher College has recently gone through a campus-wide curriculum change to rethink the meaning of a liberal arts education and how to best prepare students for life after college. As a result of this reflection, our Mathematical Reasoning general education requirement has changed into a Data Analytics requirement that requires students to take one semester-long course learning the foundations of data analytics and then another semester-long course learning data analytics techniques in the context of another discipline, usually their major. With a desire for the first semester of calculus to fulfill the first of these requirements, to reduce the course load for STEM students, and to make the calculus sequence more relevant and useful to students, we resequenced and redesigned all three semesters of calculus. This talk will focus on the new first semester course, Calculus Through Data and Modeling, which includes an applied focus, single and multivariable topics, data literacy, and computational work in RStudio. It will also give an overview of how the first year of this new approach has impacted Goucher students and the new curriculum.

**Salam Turki** Rhode Island College
**Houssein El Turkey** University of New Haven
**Yasanthi Kottegoda** University of New Haven

### Implementing preclass readings in Calculus

Active learning practices highly depend on students' preparation for class in advance. However, reading Calculus can be a challenging task to students. We address this concern by assigning targeted pre-class readings in multiple Calculus courses. We report on two implementations, provide students' feedback, and discuss the lessons learned from these implementations.

**Peter Olszewski** Penn State Behrend

### Boot Camp for Freshmen Calculus I Students

In the fall 2017 semester, Penn State Behrend launched a new foundations course emphasizing study skills and review basic mathematical principles. The major goals of this six-week course are to help college freshmen recall fundamental algebraic and trigonometric skills needed for Calculus I. In order to be placed into Calculus I, students must take an online test, ALEKS. For this boot camp class we targeted students who received scores at the lower end of the Calculus I placement range. In this presentation, I will outline how I taught the course using Knewton homework software and the overall results of this pilot study.

**Aaron Trocki** Elon University
**Karen Yokley** Elon University
**Jan Mays** Elon University
**James Beuerle** Elon University

### Strategies that Support Students Meeting the Demands of a First-Year Calculus Course

First-year calculus students' previous encounters with calculus do not guarantee success at the college level, which represents different expectations for demonstrating understanding often using new technologies. To address this situation, four calculus professors researched pedagogical strategies to meet the learning needs of first-year calculus students including those from diverse backgrounds. We employed three specific pedagogical techniques to enhance comradery, collaboration, and conceptual understanding of calculus to promote the success of all students. These techniques were used in combination to get to know students at the beginning and throughout the semester, utilize student discourse, and formatively assess understanding. Our research team considered the effectiveness of strategy implementation from the student perspective using domains defined by Schoenfeld (2016): 1) access to mathematical content; 2) agency, authority, and identity; and 3) flexible instruction that responds to students' needs and thinking. Data was gathered from students in four sections of a first-year

calculus course using pre and post-questionnaires and from their professors in the form of reflections and documented strategy use. Student and professor reactions to priming the calculus pump with these pedagogical strategies will be shared along with recommendations on how to best implement in future classes.

**Della Dumbaugh** University of Richmond, Richmond, VA

#### Moving Calculus from the Classroom to the Boardroom

With an increasing interest in workforce preparedness for our students, our early general education requirements have never occupied a more important role in the college curriculum. For many students, the actual *content* of Calculus I is not necessarily a requirement for success in their professional or personal lives. But the *process* of working through and solving challenging problems that involve these mathematical skills can prove immensely valuable in the long run. The Calculus classroom can serve as an ideal platform to use mathematical skills as a segue to discussions about the growth mindset, inclusivity, and social choices, among others. Our faculty can Teach specific mathematical skills, Identify broader uses of those ideas, and help our students Learn ways to Transfer those skills to personal and professional ventures through a process succinctly described as TILT (your students forward). This talk offers tips for how to make the TILT experience a daily part of the Calculus I classroom.

## Part B: Friday, August 3, 1:30–5:25 PM, Governor's Square 16, Plaza Building

**Robert Sachs** George Mason University

#### Reconceptualizing the Integral and the Fundamental Theorem

Integration as a tool to create the average value of a continuous function is an alternative approach that has worked well for me. It addresses known student difficulties with the "area under the curve" traditional introduction. Students can build the notion of a Riemann sum of products and relate it to the difference quotient. The use of a variable endpoint then seems more natural as the corresponding difference quotient was used earlier with a moving point to define the derivative. An added benefit is that this conceptualization works well in higher dimensions for those continuing to multivariable calculus.

**Melissa Lindsey** Dordt College

#### Interleaving Derivative Rules and Applications in Calculus I

Calculus I courses are typically organized in such a way that after students have been introduced to the concept of a derivative they spend several weeks learning all of the derivative rules followed by several more weeks learning about applications of derivatives. There is a very obvious reason for this default - it is the way that most calculus textbooks organize the material. At the end of the Fall 2017 semester I found myself wondering if that arrangement of course content was ideal for student learning. After encountering the idea of interleaving discussed in the book 'Small Teaching: Everyday Lessons from the Science of Learning' by James M. Lang I decided to interleave the derivative content in my Spring 2018 Calculus I course. Interleaving involves spacing out learning over time and strategically incorporating the practice of skills you are seeking to develop. In my calculus course interleaving meant that instead of covering the content in the traditional way, I reorganized the content so that students alternated between learning a derivative rule and learning an application of the derivative. The logistics of implementing such a course design will be discussed followed by an overview of the benefits of such a design to students - both those who had previous experience with calculus and those who were encountering the content for the first time.

**Jessica Kelly** Christopher Newport University

#### Enhancing a First-Year Calculus Course with Mathematica Assignments

The use of computing software such as Mathematica, Maple, or MatLab in a first-year Calculus course can allow students to develop intuition for concepts without tedious calculations. Exposure to a computing software provides students an opportunity to engage with material in a hands-on and visual environment. Additionally, students are able to learn the basics of a new programming language. As part of a first-semester calculus course, I developed a series of Mathematica labs specifically designed to enhance course lectures. These labs serve as an inquiry-based exploration of calculus concepts such as the Squeeze Theorem, Intermediate Value Theorem, Riemann sums, and area between curves. No prerequisite knowledge of Mathematica is required for students, and although the calculus concepts are the main focus of the labs, students are gradually introduced to new functions and commands of Mathematica throughout the series. In this talk, I will discuss the creation of these labs; in particular, the scaffolding of assignments to teach both calculus concepts and Mathematica commands, the incorporation of results from these Mathematica assignments into course lectures, and practical implementation aspects.

**Jiyeon Suh** Grand Valley State University

### First Year Calculus with Python Coding

Michael Dorff (BYU) gave a presentation "Careers in Mathematics" to GVSU math majors in Oct. 2016 in which he explained that employers suggest students do the following: 1. Learn to code, 2. Develop communication skills, 3. Do a research project, 4. Learn about other disciplines. We would like to share our experience of a trial to address the above points in a first-year Calculus course in Fall 2017 with the following components: 1. Examples of data introducing the concepts, 2. Related data analysis and problem-solving through coding with Python, 3. Team project on interdisciplinary topics resulting in a presentation and a paper. We will present a selection of examples used in the course and explain how the basic Python coding was introduced and utilized. We hope to share some reflection and analysis of this trial as time permits. We also invite ideas and suggestions from the community for the improvement in this endeavor to enhance students' Calculus learning experience.

**Robert R. Rogers** SUNY Fredonia

### We Integrate Differentials, Not Functions

The typical integral calculus course starts with limits of Riemann sums of functions, even though the history of adding infinitesimals (differentials) and the Fundamental Theorem of Calculus predates Riemann by 130 years. We will talk about how these topics can be rearranged to better reflect their historical development and to better serve the pedagogical needs of our students by solving problems with calculus before delving into the foundational aspects of Riemann sums.

**John Rock** Cal Poly Pomona

### RIP: Row Integration by Parts

Some faculty and students have heard of the tabular method for integration by parts, often seeing it as nice trick for certain special cases. However, these people don't know RIP! (Pun intended.) The tabular method is not as limited as its reputation suggests. By constructing the table with the emphasis placed each *row*—and therefore, with each new integral—the tabular method becomes an efficient bookkeeping technique for any application of integration by parts. This talk features several examples showcasing the utility of the technique, referred to as 'row integration by parts' or simply 'RIP', and includes the tic-tac-toe example made famous by the film *Stand and Deliver*. Additionally, the RIP method allows for an elegant derivation of Taylor's Formula with integral remainder and other major results in analysis. The RIP method is both easy use and easy to learn. A worksheet will provided for members of the audience who would like to take a RIP at the technique themselves!

**Marshall Ransom** Georgia Southern University

### Logs in Calculus and Maybe More.....

Exponential and logarithmic functions are fundamental in mathematics. Applying basic calculus to an exponential function of a single, real variable allows us to connect these two functions *without* defining the natural log as an integral. Most calculus texts simply define the natural log function as an integral. This does not need to be a definition. All properties of logarithms follow from this work.

**Eric Miles** Colorado Mesa University

### Estimating Pi as an Introduction to Limits in Calculus I

In the first three days of Calculus I, I guide my students in estimating pi using approximations to the circumference of the unit circle. This extended activity serves to introduce the idea of a limit, and more holistically, to introduce the flavor, beauty, and difficulty of Calculus. In this talk, I'll walk through how my class gets from "How could we possibly approximate pi?" to a Google sheet that uses an intimidating formula (a formula that they built!) to get successively better approximations. I'll also mention how I handle Precalculus review and order of topics following the first week.

**Jack Bookman** Duke University
**Michael Jacobson** University of Colorado Denver

### A Necessary Condition for Priming the Calculus Pump: Preparing Graduate Students to Teach

Any effort to improve the instruction of first year calculus must address the roles played by graduate students. This assertion is supported by the research done in MAA's national study of college calculus (NSF Award # 0910240) which identified seven characteristics of successful calculus programs, one of which is the effective preparation of graduate teaching assistants for their teaching responsibilities. Effective preparation will not only improve the teaching graduate students do during their years of graduate study, but it will impact the teaching they do in the next 30 + years at a myriad of institutions around the country. In this session, we will present the recommendations developed at an NSF sponsored conference (NSF Award # 1654273) that included department chairs, representatives of professional organizations and experienced providers of this professional development (PD) to graduate students and scholars who conduct research on the teaching of undergraduate mathematics. In particular, three major recommendations will be discussed: 1. An understanding of instructional approaches that foster student engagement. 2. Knowledge of student thinking and learning. 3. Sustained development for teaching. We will also discuss

how the resources provided by the CoMInDS project (NSF Award # 1432381) can support the structural efforts by departments to improve the PD of graduate students and, in particular, to positively impact the teaching of first year calculus.

**Eugene Boman** Penn State, Harrisburg Campus

#### Differentials, Not Derivatives

Differentiation rules predate the derivative by more than 100 years and computing differentials is pedagogically appealing to students. Moreover differentials incorporate the chain rule, implicit differentiation, related rates, and other calculus topics very naturally and intuitively. We can and should be emphasizing fluency with and application of differentials as a first step toward understanding and using derivatives before the more foundational limit concept is introduced. See how a typical University Calculus I class can be modified to do this.

**Eric C. Johnson** U.S. Coast Guard Academy

#### Math Placement at the Coast Guard Academy

Prior internal studies at the Coast Guard Academy (CGA) concluded that how cadets perform in their freshman fall math course is the best predictor of not only whether those cadets successfully complete their first year, but also whether they graduate at the end of four years. The CGA Department of Mathematics has developed our own Math Placement Program in an effort to maximize each incoming student's chances for success by making sure s/he is placed in the most appropriate initial math course. Having served as the Math Placement Program coordinator for eight years, in this talk I'll explain our Math Placement process, present a longitudinal comparison of our two placement models, a comparison of entering students' AP Calculus scores vs. our validation exam scores, and discuss the overall effectiveness of our Math Placement program.

**Adam Castillo** Florida International University
**Charity Watson** Florida International University
**Geoff Potvin** Florida International University
**Laird Kramer** Florida International University

#### Implementing Modeling Practices in Calculus at a Hispanic-Serving Institution

Mathematical modeling - using mathematics to represent, analyze, and provide insight into real-world phenomena - is a lens for supporting active learning in mathematics classrooms, particularly in the teaching and learning of calculus. However, new approaches to calculus that improve student outcomes at the postsecondary level are understudied. This talk will describe the first pilot year of a multi-year study that is being conducted to bring the authentic practices of mathematicians and mathematical modeling into the classroom, which include: students actively working in groups to develop modeling and problem-solving skills; a culturally responsive learning environment that features multiple representations, argumentation and fostering constructive perseverance; and building proficiency with mathematical terminology, language constructs and symbols. The pilot intervention involved the use of a studio-based approach complimentary to mathematical modeling, in which students learn content and the practices of mathematicians by actively doing mathematics in a lecture-reduced classroom with a focus on modeling processes themselves and the discourse employed by students working with peers. We will present a summary of the impacts of this pilot intervention on student outcomes and will also discuss the ongoing, cyclical process of evaluating, modifying, and implementing curricular materials and research instruments.

## Part C: Saturday, August 4, 9:00–11:35 AM, Governor's Square 16, Plaza Building

**Stephanie L. Fitch** Missouri S&T
**Paul N. Runnion** Missouri S&T

#### Personalizing Placement with a Multi-Faceted Approach

At Missouri S&T, the majority of our incoming freshmen arrive having had some sort of experience with calculus. On our placement exams, many of these students - even those with credit for calculus - demonstrate a concerning lack of mastery of the foundational algebra and trigonometry skills required for further calculus coursework. As part of our placement process for incoming students, we offer intense workshops during the freshman orientation week to address some of these prerequisite deficiencies. At the end of the workshops, students retake a placement examination, giving us an opportunity to modify their placement prior to the start of classes if needed while also giving us clear pre- and post-test data on the workshops. We will discuss our overall placement process, workshop structure, and share associated student success data.

**Jennifer Elyse Clinkenbeard** CSU Channel Islands, CSU Monterey Bay
**Cynthia Wyels** CSU Channel Islands

#### Connected With Calculus: Building Community Online

We present the design and implementation of an online forum to provide access to a community of individuals taking, teaching, and otherwise engaging with Calculus I material. Using the Slack messaging platform, calculus students post questions, comments, responses, and resources for one another. Students have the opportunity to contribute to the community through mathematical conversation. At the same time, students build proficiency and confidence in prerequisite and current material by posting videos, resources, and explanations. Because Slack is both a mobile and desktop app, students have the opportunity to engage with each other, and with calculus, in any time or place. We discuss themes that emerge in student posts and reflection assignments, indicators for student engagement level in the online community, and correlations between online student activity and key student outcomes. Session participants will also be provided with practical information regarding the set up, maintenance, and assessment this type of forum.

**Trefor Bazett** University of Cincinnati

#### The Effect of Flipping Calculus on Attitudes, Behaviors, and Performance

This session will discuss a series of easy-to-implement, evidence-based engagement strategies used both in class and pre class to empower our students as they learn (or relearn) Calculus. Twelve sections of first semester Calculus used a flipped classroom model, while a further eleven sections used traditional lecture, with common homework and tests. A major focus of the flipped classroom model was on faculty sharing in the creation of engaging, pre-class video modules designed to emphasize a personal connection to the instructors and reduce cognitive load on the students. These modules empowered students to embrace in-class problem solving sessions that emphasized deep conceptual understanding. The 1050 total students received pre and post concept and attitude inventories as well as submitted an engagement survey, and classes were observed with COPUS. As compared to prior flipped classroom efforts using publisher-created materials, engagement metrics more than doubled, with even stronger engagement results among more at-risk student populations. Additionally, flipped classrooms had a significant advantage on exam performance and DFW rates, while the effect on attitudes towards mathematics was mixed.

**Daniel Watson** Mississippi College

#### A Flipped Classroom Approach to a Summer Calculus Course

A flipped classroom approach is implemented in a summer Calculus course in an attempt to transition from a passive style of learning to an active style of learning. The overall goal is to improve student understanding, performance, and critical thinking skills. Homework consists of reading an assigned section in the textbook and/or instructor online notes, completing a summary of the concepts including an interpretation of any definitions and theorems in the student's own words, and asking at least one question that the student did not understand from the reading. In the classroom, the instructor addresses the questions submitted and engages students in a discussion of the material that the students read the night before. The remainder of the class consists of group problem sets with the instructor available for help. The scores on the final exam will be compared to previous summer Calculus courses as well as student evaluations to draw conclusions on the effectiveness of this approach.

**Ann Marie Harmon** Brigham Young University - Idaho

#### Calculus - Twice Flipped

A physics teacher once asked me, "What are you guys teaching over there in Calculus? My students need to do derivatives and integrals by week three and they're still doing limits!" With that in mind, I divided my calculus course objectives into the HOWs and the WHYs. In the HOWs, we learn the mechanics of derivatives and integrals along with simple word problems types. In the WHYs, we learn the mathematics behind those techniques. The structure of the class is helical so that topics are revisited multiple times for deeper understanding. In addition, I flipped the classroom so that the students prepare mundane tasks before they come to class and we use the class period to delve more deeply into the topic. I would like to share how I do it and report the classroom research I did to answer the question, "How did my students in Calculus I perform in the subsequent course compared to the rest of the Calculus I students on campus in the last 5 years?" The results were significant. Hence, we have Calculus - Twice Flipped.

# Ready or Not: Corequisite Courses and Just-in-Time Review

## Friday, August 3, 1:30–5:05 PM, Governor's Square 17, Plaza Building

**Organizers: Rebecca Hartzler** University of Texas at Austin
**Suzanne Doree** Augsburg University
**Frank Savina** University of Texas at Austin

Many students enter college not yet ready for college-level mathematics. For some, embedded just-in-time review can fill in gaps, but for underprepared students many colleges and universities are showing greater success with "corequisite" courses to increase completion, especially for underserved populations. Talks describing the curricular change process are welcome, especially examples of successful responses to rapid large-scale implementation requirements.

**Francisco Savina** The University of Texas at Austin

### Co-requisite Courses: The Right Math at the Right Time

Across the country, institutions are seeing large-scale, significant gains in student success thanks to the aggregated wisdom of over two decades of research and practice in developmental education and gateway course-taking. In this session, participants will explore the connection between co-requisite courses and *gateway momentum*, a measure of near-term progress that can predict long term success and allow institutions to gauge effectiveness of reforms much earlier. Participants will also gain an understanding of key considerations required to scale co-requisite courses, and an understanding of opportunities to leverage co-requisite models to generate *program momentum* for both STEM and non-STEM students.

**Lori McCune** Missouri Western State University

### Corequisite Implementation at Missouri Western State University

In the state of Missouri, the Department of Higher Education created a Corequisite At-Scale Taskforce (CAST) with representatives from each of the state colleges and universities to encourage implementation of corequisites at scale in Missouri institutions. As a member of this taskforce, I will describe the work of CAST and share the success of corequisite implementation at-scale in the quantitative reasoning course at my home institution, Missouri Western State University. I will describe our corequisite lab and share institutional data demonstrating student success.

**Alison Reddy** University of Illinois

### Corequisite College Algebra at Illinois

Half of all students starting college are enrolled in remedial mathematics courses at two-year institutions, and one-quarter at four-year institutions. Most institutions offer a spectrum of introductory mathematics courses and often students are not sufficiently prepared for the most basic offering. At the University of Illinois, this course is College Algebra, where half of the students enrolled in the course lack a sufficient mathematics background for direct course placement. The solution at Illinois is to offer a corequisite course instead of creating a new prerequisite course. Embracing the implementation of corequisite support is a challenge and the goal of the program is to maximize student success within the context of a single course. Corequisite support acknowledges the mathematical knowledge of students while simultaneously strengthening their mathematical knowledge. An additional challenge is that students enter College Algebra with very diverse mathematical needs and mathematical background knowledge ranging from arithmetic to almost Precalculus and yet are expected to demonstrate mastery of the same material during the semester. Meeting students where they are is additionally complicated by the large diversity of mathematical backgrounds of typical populations of new students: the majority (approximately 70%) of College Algebra students at the University of Illinois are first generation college students or underrepresented minority students. Initial data indicates that the implementation has been very successful in meeting the individual needs of all students, evident through improved success rates and increased student satisfaction, retention rates and success rates. Collected data will be presented.

**Mary B. Walkins** The Community College of Baltimore County
**Jesse Kiefner** The Community College of Baltimore County

### Get AMPed about Corequisite Courses

In 2009 The Community College of Baltimore County (CCBC) developed a Corequisite Mathematics Model called the Accelerated Mathematics Program (AMP). This initiative was designed to confront dismal success and retention rates in developmental and credit level mathematics courses. After faculty members are trained, two sequential mathematics courses are dovetailed, and students experience a holistic blend that enables them to complete their college mathematics requirements at a quicker pace. Longitudinal data analysis indicates that students are twice as likely to succeed in mathematics when electing an accelerated path. This presentation will highlight the evolution of AMP at CCBC and discuss data analysis, course structure, faculty training, scaling up, future planning, and instructional strategies.

**Andy Richards** Central Washington University

### Helping Developmental Students Enter into College Level Mathematics Courses

Too many students are entering college unprepared for college-level mathematics. Central Washington University has two programs established to support students in pre-college level mathematics. One program is a co-requisite Leap program designed to accelerate motivated students' entry into pre-calculus. The other program is a Math Bridge Program designed to increase the number of incoming students who place into college level mathematics their first term at CWU. This paper will discuss both programs and discuss other efforts to address those students placing into developmental level mathematics courses.

**Kenneth A. Parker** NYC College of Technology

### College Algebra and Trigonometry *Enhanced*: A Co-Requisite Model with "Lab-Style" Explorations

This talk will showcase a "lab-style" approach to the co-requisite model, engaging students in weekly, technology-enhanced explorations that are designed to support our existing college algebra and trigonometry curriculum. In our model, students are grouped together to explore geometric objects such as lines, parabolas, and circles, while simultaneously attempting to construct their corresponding algebraic equations. Students are then required to complete lab reports, describing their process of discovery any conclusions about the relationships between the algebraic and geometric objects being explored in the lab. These lab-style explorations were designed in GeoGebra and have been integrated into our existing WeBWorK installation, offered at zero-cost to students, and are freely available to any other institutions using WeBWorK. Initial results from our first year of implementation are very promising, showing significant gains in many of the areas specifically targeted by these lab-style explorations. This work was supported by *Opening Gateways to Completion,* a U.S. Department of Education Title V collaborative grant between New York City College of Technology and Borough of Manhattan Community College, CUNY.

**Emily Gismervig** University of Washington Bothell
**Cinnamon Hillyard** University of Washington Bothell

### Peer Facilitator Led Support Courses for Precalculus

In Fall 2017, we piloted a program where undergraduate peer facilitators (PFs) led support classes that accompanied our Precalculus II course. Each PF met weekly with a group of 15 precalculus students to review material from that week's Precalculus II class, to review prerequisite course material and to discuss college readiness skills. Enrollment in the support course was mandatory. Additionally, Precalculus II faculty and the PFs identified at-risk students on a weekly basis so that the academic advisor on the pilot team could intervene with these students as early as possible. The precalculus sequence has been an ongoing source of concern at UW Bothell. Students who do not do well in Precalculus II cannot complete calculus and apply to STEM and business majors. This is a concern for a large number of UW Bothell students: 45% of this year's first-year class and nearly 100% of students taking Precalculus II indicated an interest in a STEM or business field. After revamping the precalculus curriculum for the 2016-17 academic year, only 70% of students earned high enough grades to move on to Calculus I. When we implemented the peer facilitator program in Fall 2017, 78% of Precalculus II students received high enough grades to move on to Calculus I. I will detail changes that were made between the Fall, Winter and Spring Quarters, provide data on student performance in Precalculus II, and discuss how we implemented the program and challenges that we faced.

**Eileen C. McGraw** Stevenson University
**Sarah G. Blanset** Stevenson University
**Thairen G. Dade** Stevenson University

### The Journey to Co-remediation

Students at Stevenson must satisfy a quantitative literacy requirement for graduation. Most do this through one of three classes (in order of enrollment): Quantitative Reasoning, Introduction to Statistics, or College Algebra. Our developmental mathematics class had originally been designed with only College Algebra in mind. First we redesigned the course to match the requirements for the courses satisfying the new quantitative literacy requirement, instead of just College Algebra. Then we created a two semester "stretch" version of the Quantitative Reasoning course, allowing students to earn some credit towards their degree in their first semester. We decided on a different approach for our Introduction to Statistics: what we called a "just in time" remediation course that would be taken concurrently. The first results are in, and are good enough that we are cancelling the stretch course and developing co-remediation courses for all quantitative literacy courses.

**Rachel Weir** Allegheny College
**John Meier** Lafayette College

### Just-In-Time Mathematics Support Using Online Modules: Findings from a Multi-Institutional Project

Allegheny College and Lafayette College were two of the ten partner schools in the 2014-18 FIPSE (Fund for the Improvement of Postsecondary Education) project "Developing Just-In-Time Mathematics Support to Increase STEM Completion," funded by the US Department of Education. This project was led by Bryn Mawr College and involved integrating online modules intended to address gaps in students' computational abilities in introductory calculus, chemistry and physics courses. In this talk, we will provide details about the module implementation and share successes, both at our individual institutions and across the partner schools. We will also discuss the many lessons learned regarding crafting effective modules and the difficulties of implementing across multiple institutions.

**Samuel Luke Tunstall** Michigan State University
**Becky Matz** Michigan State University

### Meeting students where they are: Supplemental instruction in large-scale quantitative literacy courses

Abstract: As colleges and universities enroll an increasingly diverse student body, courses in developmental and introductory mathematics are changing. In particular, there is a need for institutions to provide pathways through college level math courses that equitably meet the needs of students with a wide array of incoming mathematical knowledge and skills. In light of salient questions about pedagogy, pass rates, and effects on degree completion time, some institutions have moved away from requiring students to enroll in non-credit bearing remedial mathematics courses. At Michigan State University, for example, courses in Quantitative Literacy (MTH 101 and MTH 102) now directly enroll students who previously would have placed into Developmental Mathematics. This shift in requirements has been supported by changes in course structure and content; in particular, some course sections now include an extra day of instruction to help students bridge gaps in their requisite skills. An intuitive approach to supporting student learning, essentially increasing time on task, these course sections will be examined in this study for their efficacy in improving both course grades and persistence for students. In this talk, we will share information about this approach, as well a quantitative, post-hoc analysis on both current student populations and comparisons to historical cohorts of similar students.

**Nicholas Shay** Central Ohio Technical College

### Using Corequisite Remediation to Overcome Barriers in Technology

Undergraduate courses in statistics, linear algebra, and calculus often explore problems using software such as Excel, Matlab, Minitab, R, or traditional graphing calculators. Many students have little or no experience with these programs prior to enrolling. This lack of exposure leads to learning barriers in mathematics. The just-in-time remediation model affords instructors the opportunity to familiarize students with software basics in a classroom setting, encouraging students to fully engage with the concepts of the course, and ultimately gain a better understanding of the material. This presentation will detail the use of co-requisite learning in an Introduction to Statistics course that teaches the basics of Excel, as well as graphing calculator technology. Finally, the presentation will explore how the concepts apply to other undergraduate mathematics courses.

**Kathy Andrist** Utah Valley University

### Making Waves in Math Placement

Traditionally, about 75% of students begin their UVU math pathway in developmental mathematics courses and despite careful placement and enforcement of pre-requisites, the failure rates in these courses are unusually high (40%+). Why then, do students perceive themselves as being placed into unnecessary math courses which are filled with content already studied in High School? Something doesn't add up. Details of how this large, open enrollment, public university is making waves by rethinking and redesigning the placement and review process to improve math placement, expedite remediation and increase student success. Preliminary data suggests that students who were once sinking are now swimming in math success.

## Recreational Mathematics: Puzzles, Card Tricks, Games, Gambling and Sports

### Part A: Friday, August 3, 10:30 AM–12:25 PM, Grand Ballroom II, Tower Building

**Organizers: Paul R. Coe** Dominican University
**Sara B. Quinn** Dominican University
**Kristen Schemmerhorn** Concordia University Chicago
**Andrew Niedermaier** Jane Street Capital

Puzzles, card tricks, board games, game shows, gambling, and sports provide an excellent laboratory for testing mathematical strategy, probability, and enumeration. The analysis of such diversions is fertile ground for the application of mathematical and statistical theory. Solutions to new problems as well as novel solutions to old problems are welcome.

**Eric Eager** University of Wisconsin - La Crosse
**George Chahrouri** Pro Football Focus

### Using Advanced Accuracy Data and Machine Learning to Model Quality of Play at the Quarterback Position

In this talk we introduce a novel approach to charting and analyzing quarterback play via advanced accuracy charting and machine learning. We show how stable and predictive various aspects of quarterback play are using this data, and demonstrate how these analyses can differ from traditional methods in its derivation of quarterback quality. We discuss the 2018 draft class of quarterbacks through this lens.

**Paul von Dohlen** William Paterson University

#### Building a Numerical Baseball Simulator

The fact that the game of baseball can be seen as a sequence of discrete, repeated situations makes it very suitable for numerical simulations. And because of the large number of repeated occurrences, a wealth of statistics has been collected from games. In recent years, virtually every action occurring on a baseball field has been logged and cataloged presenting the opportunity for baseball to be modelled via a simulator. In this talk, we will discuss the historical development of baseball simulators, the proliferation of statistical resources, the challenges faced in creating a realistic simulation as well as my work on a baseball simulator. We will also discuss the opportunities for undergraduate research as this project has involved and continues to involve undergraduate students since it incorporates aspects of data collection and analysis, linear algebra, probability, statistics and numerical computation which are accessible to undergraduate mathematics majors.

**Jeff Poet** Missouri Western State University

#### Tournament Scheduling Improvements

While attending a recent quizzing event, shortcomings were observed in the schedule. In this tournament, teams compete three at a time over three days with a two-day round robin portion followed by an elimination round that leads ultimately to a championship round. This is a 30-year-old tournament with the number of teams varying between 20 and 61 over that span. To my knowledge, this is the first time a mathematician has looked at this applied combinatorics problem. This talk outlines the flaws of the current system and suggests improvements to the scheduling structure.

**Thomas Q. Sibley** St. John's Universtiy

#### Fantasy on a Baseball Theme

There are paths along edges of some polyhedron, such as a cube, that mimic the seam of a baseball: the path splits the surface into two congruent regions that fold flat. We pursue ideas that come from this unusual way of approaching polyhedra. Such paths provide connections with other mathematical ideas, such as Hamiltonian paths, Euler's formula and symmetries.

**Benjamin Wilson** Stevenson University

#### Would Wheel of Fortune be Easier in Dothraki or Klingon?

Using concepts from information theory such as entropy, we analyze the complexity of several constructed languages. First defined and studied by Claude Shannon in 1948, the entropy of a written language measures how much information is produced on average for each letter of text in the language. Shannon estimated the entropy of written English by doing experiments to approximate word, letter, and n-gram frequencies. We analyze and compare the entropy and other similar quantities of constructed languages such as Dothraki (Game of Thrones) and Klingon (Star Trek). By analyzing the complexity of several languages we can decide which would lead to the easiest game of Wheel of Fortune assuming the player is fluent in all of them.

**Anthony DeLegge** Benedictine University

#### Beyond the "Monty Hall Problem": The Mathematics of *Let's Make a Deal*

The "Monty Hall Problem," originally posed in a 1990 issue of *Parade* magazine, has captivated mathematicians and their students for over 25 years. But, there is a problem: the game depicted in the "Monty Hall Problem" was never actually played on the game show it is supposedly based on, *Let's Make a Deal*! There are, however, numerous games that are played on the show that, much like this famous problem, involve a little trickery with probability that can get the players (and viewers) to second-guess their decisions. This talk will detail some of those games, with plenty of audience participation.

## Part B: Friday, August 3, 1:30–5:15 PM, Grand Ballroom II, Tower Building

**Arthur T. Benjamin** Harvey Mudd College

#### Mathematical Card Tricks

The presenter will demonstrate and explain some amazing mathematical card tricks.

**Jang-Woo Park** University of Houston-Victoria
**Ricardo Teixeira** University of Houston-Victoria

#### New Card Trick: "Predicting the Finalists"

In this talk, we will brief review two famous mathematical ideas used in various magic tricks: Si Stebbins and Josephus problem. Then we will apply them to a new card trick where a random process of selecting the finalists of a two-league tournament can be predicted when a volunteer randomly cuts a deck of cards into two (creating the different leagues).

**Robert Wolverton** US Air Force Academy

### War, What is it Good For?

The card game *War* is a child's game. *War* introduces young people to a deck of cards and reinforces some basic facts about numbers. Game play is random and suffers from a few deficiencies. First, a single game averages 15 - 20 minutes, and children often lose interest in the game. Second, the random game play is highly repetitive and boring. To reverse these shortcomings, we introduce a minor modification to the rules. This modification creates a fast paced and dynamic game which retains all the simplicity of the original version. Moreover, some interesting mathematical results appear when the game play is analyzed. We look at winning percentages based upon a variety of strategies as well as the interesting results associated the length of game play.

**Jathan Austin** Salisbury University

### Using Games as a Context for Mathematical Modeling

Games can be used as a vehicle for exploring many mathematical topics. In this presentation, we explore how designing a gameplay strategy for the card game "No Thanks!" can be used as a course context for engaging in mathematical modeling.

**Robert W. Vallin** Lamar University

### Penney's Game with Strange Coins

Penney's Game is a non-transitive game involving tossing a coin. Each player chooses a three-outcome sequence. A coin is flipped until one of the player's sequences arises. This game has been thoroughly studied with a fair coin. In this talk, we look at some ways to determine odds and wait times with Penney's game, and look at the effect of using a weighted coin.

**David McCune** William Jewell College
**Lori McCune** Missouri Western State University

### Markov Chains, Your Children, and You

A *Markov chain* is a probabilistic tool that can be used study a variety of real-world situations. Arguably the most entertaining application of Markov chains is to calculate probabilities in games where much (if not all) of gameplay is left to chance. Markov chains have been used to analyze the probability of landing on given properties in *Monopoly* and to calculate the expected length of a game of *Chutes and Ladders*, for example. Similar to these games, the game *Count Your Chickens!* involves the movements of a token on a game board where the movement is determined by chance, and thus this game can be analyzed using Markov Chains. In this talk we will discuss the use of Markov chains to compute win probabilities for the original *Chickens* game board and for variants of the game in which we change the structure of the board.

**Thomas J. Clark** Dordt College

### Can Camels Compute Conditional Probability? An analysis of *Camel Up*.

In the popular board game *Camel Up* players score points by betting on five camels that move around a track under the whims of a set of dice. While the game is fun to play on its own merits, winning the Spiel des Jahres in 2014, it affords an interesting mathematical analysis. In this talk, we explore the game from a pedagogical perspective laying out ways in which the topics of conditional probability and expected value can be taught in a concrete and compelling context using this game. Secondly, we explore whether using a computer to calculate the probabilities of possible outcomes and their expected values can give a player an advantage in placing "optimal" bets.

**Joseph M. DiMuro** Biola University

### The Warden's Game: An Application of de Bruijn Sequences

Consider the following game between a prisoner and a warden. There is a row of coins on a table; some show heads, and some show tails. If the rightmost coin shows heads, the prisoner moves it to the far left, and optionally flips the coin. If the rightmost coin shows tails, the warden moves it to the far left and optionally flips it. The prisoner earns his freedom if all the coins show tails. Can the prisoner win this game? In this talk, we'll see how de Bruijn sequences play a role in the optimal strategy for this game. We'll also look at a variation of this game, played with n-sided dice.

**Stephen Andrilli** La Salle University

### The Vanishing Square Puzzle and the Fibonacci Sequence

The classic "vanishing square" puzzle is one in which a unit square of area seemingly disappears from the union of four figures –two triangles and two hexagons –on a coordinate grid of the plane after the four figures are rearranged upon the grid without deformation. The solution to this geometric mystery involves a fundamental algebraic property of the Fibonacci Sequence. After presenting an algebraic proof of this property – involving products of successive (as well as nearly successive) members of the Fibonacci Sequence – we find that the original geometric puzzle provides a convenient illustration of this result. In fact, it forms a particularly effective example to demonstrate that many properties of this sequence have both an algebraic and a geometric interpretation which complement each other.

**Edmund A. Lamagna** University of Rhode Island
**Robert A. Ravenscroft** Jr., Rhode Island College

#### Sum Fun with Fibonacci and Friends

The Fibonacci numbers have for centuries been the subject of many explorations in recreational mathematics. One well-known result is that the sum of the first $n+1$ Fibonacci numbers can be expressed in terms of a Fibonacci number, $\sum_{k=0}^{n} F_k = F_{n+2} - 1$. In a similar way, the sum of the first $n$ harmonic numbers can be expressed in terms of a harmonic number, $\sum_{k=1}^{n} H_k = (n+1)H_n - 1$. The sequences $\langle F_k \rangle$ and $\langle H_k \rangle$ are examples of *linear recurrence sequences* in that each is defined by a linear recurrence relation with constant coefficients. $F_k = F_{k-1} + F_{k-2}$ with initial conditions $F_0 = 0$ and $F_1 = 1$ in the case of the Fibonacci numbers, and $H_k = H_{k-1} + 1/k$ with $H_1 = 1$ for the harmonic numbers. An algorithm is developed to express sums of linear recurrence sequences in terms of the sequence name. The method is a formalization of the ad hoc technique used to evaluate the sum of the Fibonacci numbers. The algorithm can evaluate the sums of all homogeneous recurrences and many inhomogeneous recurrences, and has been implemented in the Maple computer algebra system.

**Jeremiah Bartz** University of North Dakota

#### A covering property for digital root series

A digital root series is a recursive integer sequence whose recursive formula involves the modulo operation. In this talk we discuss a covering property observed when considering digital root series in certain bases and their connections with juggling sequences.

**Jay L. Schiffman** Rowan University

#### Exploring The "Reverse" Lucas Sequence 3, 1, 4, 5, 9, ...

The standard TI-84 and TI-89 graphing handhelds by Texas Instruments enter the initial terms for a recursive sequence in reverse order (second term followed by the first). This is not a problem for the standard Fibonacci sequence; for the initial two terms are both 1. On the other hand, for the Lucas sequence whose initial two terms are 1 and 3 respectively, these calculators would enter the terms as 3 and 1. If one then follows the standard recursion relation for the sequence, the output terms would correctly read 1, 3, 4, 7, 11, .... Since a vast amount of knowledge has been generated with regards to the traditional Lucas sequence, but little has been studied for what I call the reverse Lucas sequence whose first few terms are 3, 1, 4, 5, 9, ..., I proceeded to partake of a study of this sequence with regards to divisibility and periodicity patterns as well as prime outputs that have been generated. With regards to prime outputs, the number of primes is fairly plentiful among the initial one hundred terms, but becomes scarcer as we proceed. I was able to factor the initial four hundred twenty-five terms of the sequence and two of these primes containing 65 and 87 digits respectively are now featured on The Prime Curios website. Astonishing little is on the OEIS pertaining to this sequence which peaked my curiosity.

## Part C: Saturday, August 4, 9:00–11:35 AM, Grand Ballroom II, Tower Building

**Jason Rosenhouse** James Madison University

#### The Continuing Saga of the Hardest Logic Puzzle Ever

The Hardest Logic Puzzle Ever was introduced by philosopher George Boolos in 1996. You are presented with three gods: one always speaks truthfully, one always speaks falsely, and the third randomly decides whether to speak truthfully or falsely. The gods will answer any yes/no question put to them. However, they will answer in their own language, in which the words for yes and no are da and ja, but you do not know which is which. The puzzle is to determine who is who in just three questions. Since the puzzle's publication, a small industry of papers has been published analyzing its subtleties and nuances. We shall survey this work, paying particular attention to recent versions of the puzzle based on non-classical logic.

**Oscar Levin** University of Northern Colorado
**Tyler Markkanen** Springfield College

#### Knights and Knaves and Naive Set Theory

Here is a curiosity: consider the set $A = \{2, |A|\}$, where $|A|$ denotes the cardinality, or size, of the set $A$. What is that size? If it is 2, then $A$ contains just one element. But if it is 1, then $A$ contains two elements. This paradox of naive set theory reminds us of the liar paradox, "this statement is false." And just as the liar paradox inspires us to consider knight and knave puzzles, in which the reader is required to deduce which speakers are telling the truth and which are lying, so too can our paradox of naive set theory lead to puzzles we will call *puzzles of cardinality*. In this talk we will share some original puzzles of this type ranging from simple to complex. We will determine exactly which finite sets can be solutions to such a puzzle. Finally, we will see just how far we can take the analogy between our cardinality puzzles and the classic knights and knaves puzzles.

**Brendan W. Sullivan** Emmanuel College

### Open Problems in the Game of Lazy Cops and Robbers on Graphs

In the pursuit-evasion game *cops and robbers*, a team of cops and a robber occupy vertices of a connected graph and alternately move along its edges. The minimum number of cops required to catch the robber on a graph $G$ is called the *cop number*, denoted$c(G)$. This means that $c(G)$-many cops have a *winning strategy*: by following that strategy, they will certainly catch the robber, no matter how he chooses to move. Likewise, this also means that the robber has a winning strategy against fewer than $c(G)$-many cops: by following that strategy, the robber will never be caught, no matter how the cops choose to move. Analogously, we consider the *lazy cop number*, denoted $c_L(G)$, which is based on the added stipulation that *only one cop may move at a time*. In this presentation, we will share some results about lazy cop numbers of graphs, all based on joint work with the presenter and his undergraduate advisees. We hope that instructors will see that this game can be used to teach graph theory, problem-solving, and strategic thinking. We also hope to show undergraduates that they can jump in and do some research on this topic: we will pose plenty of open problems for students to work on and hopefully solve!

**David Nacin** William Paterson University

### A Spectrum of Solutions for a Set of Cyclic Groupdoku

KenKen puzzles over the cyclic group of order four have a unique symmetric cage pattern allowing for the possibility of unique solutions, though that does not mean every puzzle over this pattern can be solved uniquely. In fact, puzzles of this type can have 0,1,2,3,4,8,10, or 12 solutions. In this talk we attempt to explain what is responsible for this entire spectrum of possibilities. We examine maps between puzzles which preserve the number of solutions a puzzle has, use basic tools from group theory to reduce the problem down to a manageable number of cases, and then use ideas from graph theory to try to understand each individual situation. Graphs will allow us to visualize each case and understand why it is possible to get numbers like three and ten for the number of solutions to four by four puzzles with four clues over a group of order four. We arrive at a classification which allows us to determine exactly how many solutions any puzzle has based only on the immediate properties of the clues themselves.

**Guoan Diao** Holy Family University

### Using Games for Teaching Mathematical Concepts

No matter the subject, interest is key to truly understand and remember the content. How do we show students that math can be fun and help them develop necessary skills at the same time? Math games, of course! These games show another side of mathematics– a bright side that is obscured by the typical curriculum. In my talk I will show how to use a mathematical game, primarily KenKen puzzles, in the classroom, to teach and practice many important concepts such as factorization of an integer, the negation of logic statements with quantifiers, set operations, and even the formation of systems of linear equations. I will also present how students feel and are affected by it.

**Paul Olson** Penn State Erie, Behrend

### Evaluating Hackenbush Positions

Combinatorial Game Theory gives undergraduate students a chance to explore mathematics not usually covered in their core mathematics courses. The material is accessible and allows for creativity in their explorations. I will present results from student researchers involving the game of Blue - Red Hackenbush.

**Michael Barnsley** Australian National University
**Andrew Vince** Australian National University
**Louisa Barnsley** Australian National University

### Fractal Tiling Puzzles

We will describe some fascinating mathematical problems related to fractal tiling theory. A prize of $2000 (Australian Dollars) for proof or disproof of a simply stated tiling conjecture. We will also describe fractal chess and frango puzzle.

**Sam K. Miller** Harvey Mudd College
**Arthur T. Benjamin** Harvey Mudd College

### Challenging Knight's Tours

Over the centuries, chess enthusiasts have enjoyed the "knight's tour" puzzle. Given a standard 8-by-8 chessboard with a knight placed on an arbitrary square, the goal of the knight's tour is to move the knight 63 times such that it visits each square exactly once. A more advanced variation, the "challenging" knight's tour, requires a start square and end square of opposite color to be specified beforehand. We provide a short constructive proof that all start and end square combinations of the challenging knight's tour have a solution.

# Research in Undergraduate Mathematics Education (RUME)

## Thursday, August 2, 1:30–4:45 PM, Plaza Ballroom D, Plaza Building

**Organizers: Megan Wawro** Virginia Tech
**Aaron Weinberg** Ithaca College
**Stacy Brown** California State Polytechnic University

The goals of this session are to promote quality research in undergraduate mathematics education, to disseminate educational studies to the greater mathematics community, and to facilitate the impact of research findings on mathematics pedagogy. Presentations may be based on research in any undergraduate mathematical area. Examples include studies about students' mathematical reasoning, teaching practices, curriculum design, and faculty professional development.

**Elise Lockwood** Oregon State University

### An Initial Exploration into Undergraduate Students' Computational Activity in a Combinatorial Setting

Computational thinking and activity is becoming an increasingly important aspect of what it means to engage in scientific and mathematical work. In mathematics education, there is a need for studies that examine the ways in which students engage with computational tools as they reason about mathematical concepts. In this talk, I present results from preliminary work in which undergraduate novice programmers and counters engage with tasks designed to use basic Python programming to teach particular combinatorial ideas. I highlight both problematic issues and affordances that the students experienced in using computation in a combinatorial context. I conclude by framing this work within ongoing efforts to better understand the nature of computational thinking and activity for undergraduate mathematics students.

**Rachel L. Rupnow** Virginia Tech

### Exploring Expert and Novice Understandings of Isomorphism and Homomorphism in Abstract Algebra

Although isomorphism and homomorphism are central concepts of abstract algebra that share a focus on similarities between structures, students' emerging understandings of them require time to appropriately relate and distinguish between these concepts. As part of a larger study, class-wide surveys from two sections of a junior level course and interviews with two instructors and eight abstract algebra students are analyzed for distinctions and similarities between conceptions of isomorphism and homomorphism. Conceptual metaphors, which look at cognitive mechanisms by which people view one thing as another thing (Lakoff, G., & Núñez, R. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being.* New York: Basic Books), are used as a conceptual lens for this study. Specifically, the overlapping and non-overlapping choices of conceptual metaphors used to describe isomorphism and homomorphism are explored.

**Deborah Moore-Russo** University at Buffalo
**Monica VanDieren** Robert Morris University

### Developing a Conceptual Model for Vector Cross Products

Research on student understanding of vector cross products has focused primarily on symbolic manipulations. When graphical manipulations have been examined, they have involved static environments with paper-and-pencil activities. There is little research in post-secondary environments regarding non-contextual geometric cross product activities, especially in online contexts. This study reports on how grounded theory techniques were used to analyze the responses of 434 multivariable calculus students on items in a dynamic visualization activity situated in the free, online CalcPlot3D platform. We then compare these results with student concept map data and with results of vector studies in physics and engineering. We highlight specific aspects of cross products that multivariable calculus students tend to report and suggest categories to outline a preliminary conceptual model for student understanding of vector cross products.

**Diana L. Vasco Mora** Universidad Tecnica Estatal de Quevedo
**Nuria Climent Rodríguez** Universidad de Huelva

### Specialised Knowledge of University Lecturers of Linear Algebra in Relation to Connections

The need for research on the teachers' knowledge in relation to the teaching of algebra at the university level is recognized (Fukawa-Conelly, Johnson & Keller, 2016). The interest of our research focuses on the teacher's knowledge through a case study. With the Mathematics Teacher's Specialised Knowledge model (MTSK) (Carrillo, Montes, Contreras & Climent, 2017) we have studied the specialised knowledge of two lecturers of linear algebra. The data was obtained through video recordings and semi-structured interviews. Subsequently, a content analysis of class episodes and fragments of the interviews with lecturers was carried out. In a previous study (Vasco & Climent, 2017) we have been able to show relationships between their mathematical knowledge, knowledge of errors and difficulties of students and knowledge about examples for teaching. The aim of the current study is to deep into the teacher's knowledge on then connections of contents relative to matrices and determinants with other contents or in this same nucleus of contents. Thus, we try to approach the knowledge of this content from an advanced perspective (Klein, 1908).

**John A. Kerrigan** Rutgers University

### Productive Failure in the Undergraduate Flipped Mathematics Classroom

Over the past few years, the flipped classroom approach has been gaining popularity in higher education (Abeysekera & Dawson, 2015), particularly in mathematics (Muir & Geiger, 2015). While many studies have addressed differences between the flipped classroom and traditional methods of instruction, few have closely examined how to design activities in a flipped classroom that develop students' higher-order thinking skills (O'Flaherty & Phillips, 2015; Song & Kapur, 2017). Kapur's (2008) theory of productive failure states when students have an opportunity to generate and explore solutions to a challenging task prior to being instructed on it, they are better positioned to consolidate their knowledge during and after instruction. This mixed methods study involves two undergraduate flipped math courses at a large state University, one of which was taught using the productive failure model and the other the traditional flipped classroom model. Quantitative data from a survey and from course assessments will be used to help explain students' performance in both treatment conditions, while qualitative data from a focus group interview and video footage of in-class problem solving will be used to better understand the learning and problem-solving processes in both treatment conditions. Findings from this study will be used to help undergraduate mathematics educators design effective hybrid and flipped classroom learning environments.

**Jim Brandt** Southern Utah University
**Gretchen Rimmasch Meilstrup** Southern Utah University

### Faculty Feedback on Student Proofs

A significant body of research related to student difficulties in learning to write mathematical proofs exists. Do mathematics faculty give students consistent feedback as students learn to write proofs? In this study, a random sample of mathematics and mathematics education faculty from across the country were asked to mark up and grade a selection of student "proofs" for a given proposition from an introduction to proofs course. Similarities and differences in the faculty feedback and numerical grades will be discussed.

**Tim Hendrix** Meredith College
**Karen Keene** North Carolina State University

### Learning to Prove Through Students' Eyes: the Case of Proof by Contradiction

In the undergraduate mathematics community, we continue to be interested in how mathematics majors transition to first understanding and then developing mathematical proof. Many institutions offer a course on logic, reasoning and proof that serves as a bridge to the mathematics major. These courses are often titled some variation of *Transition to Proof*; the course in this study is *Introduction to Mathematical Reasoning*. There have been multiple research reports about this type of class, but fewer reports have been about how the students' view their learning in such a course. Data were collected from two semester-long course sections over a 16-month period. This presentation focuses on students' perceptions of the experience in such a course by considering their participation, discussion and written products concerning proof by contradiction. Students were asked to answer questions and do proof tasks in partner task-based interviews; student work products were also collected. Results will be shared by analyzing these data using open coding schemes. Results show differences in students' ability to explain proof logic conceptually and their ability to detect and correct errors in proofs. The professor's use of language was shown to be important in the students' understanding. As they developed written proofs as partners, there was clear evidence that the active learning class environment helped provide them ways to participate in the proving process together. Finally, implications for instruction that arose from studying student perceptions are shared.

**Benjamin D. Sencindiver** Colorado State University
**Mary Pilgrim** Colorado State University

### Online Homework: What Students Think and What Students Do

Online homework is ubiquitous across college calculus courses (Krause & Putnam, 2016). Student intentions and actions can be misaligned (Winne, Jamieson-Noel, & Muis, 2001). Anecdotally, perceptions around the value of and, in turn, how usage of online homework can support learning have not been discussed much in the literature. In this presentation, we will discuss student's perceptions of an online homework system, how they used it, and academic performance in a Calculus I course. Preliminary findings regarding these variables will be discussed. These findings will inform future iterations of the course to better support student's learning with and productive usage of the online homework system.

**Kristin Frank** Towson University

### Constructing Formulas from Dynamic Images: What Happens When Nothing Stays the Same?

Researchers have documented how a student's image of a situation and images of how quantities vary impacts the student's construction of formulas that describe relationships among quantities in a dynamic situation. While these dynamic aspects of one's imagery are essential, the results of this study suggest that if the student's image of the situation entails only dynamic imagery, then the student is unable to construct an invariant quantitative relationship. I provide results from teaching experiments with university precalculus students to document how students whose image of the situation focus exclusively on varying measures and sizes are limited to constructing a gross coordination of values (e.g.,

as $x$ increases $y$ decreases). I conclude by discussing the ways that dynamic digital visualizations might contribute to students' difficulties constructing and attending to constant quantities.

**Eyob Demeke** Cal State LA

**A Fine-grained Analysis of Developmental Mathematics Students' Background Mathematics Knowledge Using MDTP's Second Year Algebra Readiness Test**

In this contributed report, we present the evolution of developmental students' mathematics background knowledge after a four-week long course that emphasized active learning. The research took place at a large Hispanic serving institution in the state of California. Students' progress or lack thereof was measured using a diagnostic test developed by the Mathematics Diagnostic Testing Project (MDTP). These students were initially considered not ready for college-level mathematics coursework and were subsequently enrolled, in a four-week summer course which is designed to prepare them for a college-level math course. During each class, students would spend at least 30 minutes engaging in cooperative learning that utilizes active learning strategies such as think-pair-share, peer lesson, and wait time. A pre/posttest analysis of SYART showed that these students showed a statistically significant growth, leading us to conclude that the four-week intervention in math remediation had a considerable impact.

# Teaching Undergraduate Mathematics with Primary Historical Sources

## Part A: Friday, August 3, 1:30–4:45 PM, Governor's Square 12, Plaza Building

**Organizers: Dominic Klyve** Central Washington University
**Maria Zack** Point Loma Nazarene University
**Jeff Suzuki** Brooklyn College

In recent years, there has been an increasing interest in using primary historical sources to teach undergraduate mathematics. This approach has been used by a wide variety of faculty, including those with little background in mathematics history. This session brings together developers of materials for teaching with primary sources, instructors who teach with primary sources, and mathematics education researchers.

**Maria Zack** Point Loma Nazarene University

**Through the Looking Glass: Dodgson and Determinants**

How do you engage students with the history of mathematics in linear algebra? One answer is to engage students with a familiar computational technique done in an unusual (and efficient) way. While tutoring students in Oxford, Charles Dodgson (aka Lewis Carroll), developed a novel way for computing the determinant. The original source document where he describes his method is relatively easy for students to understand. This talk describes a multi-day class project that explores Dodgson's methods. It also discusses the student responses to learning about the area of mathematics via primary sources.

**Adam Glesser** California State University, Fullerton

**Modern Algebra—A Collection of Mathematical TRIUMPHS**

The abstract algebra sequence at CSU Fullerton primarily serves mathematics education students, as well as students concentrating in pure mathematics. As such, it is important that the course is both fundamentally enlightening and intellectually rigorous. The first course in the sequence covers basic number theory results as well as their analogues for polynomial rings. It also covers modular arithmetic and the basics of ring and group theory. Over the past year, I have been writing a book that attempts to frame the course in historical context and, at the same time, attempts to be readable by students as part of, for instance, a flipped classroom model. The book includes the writings of Euclid, Gauss, and Dedekind. Furthermore, the book contains sidenotes that help the reader learn how to read mathematics, teach the rudiments of LaTeX, and offers insight into how a mathematician thinks about the topics.

**Matthew Cathey** Wofford College

**Mathematical Communication: the Unexpected Benefit of Using PHSs**

When I agreed to test a module in my Linear Algebra class on the method for solving systems of linear equations presented in the classical Chinese text *The Nine Chapters on the Mathematical Art*, I expected my students to learn a little about the connections between ancient Chinese mathematics and modern Gaussian elimination, which would be a good introduction into the course. What I *didn't* expect is the degree to which my students struggled with the language. This created space for a dialogue which changed the way my students think, speak, and write about mathematics, as well as my approach to teaching Linear Algebra. In this talk, I'll give details on what I (and my students) gained by including this historical source in my curriculum.

**Anne Duffee** Sewanee: the University of the South

### History of Math with Only Primary Sources

In this talk, I discuss my experience teaching history of math as a discussion-based course using entirely primary sources. In particular, I reflect on structuring the course as a history of formal mathematical systems while tracing the development of certain mathematical concepts such as number and continuity. Further, I examine the significant challenges inherent to reading mathematical texts not presented in a standard symbolic notation while simultaneously defending their pedagogical importance. I conclude with concrete recommendations for instructors who plan to use historical sources within their courses.

**Shawna Mahan** PPCC

### Creating Calculus Projects Using Primary Sources

John Wallis's body of works have a wealth of algebraic, calculus, and analytical material to source. Discuss the discovery process of creating calculus projects using Wallis's *Arithmetica Infinitorum* (1656) and *De Algebra* (1685). The projects' purposes are to help students to consider primary source material, to strengthen understanding of prior or current calculus topics, and to diversify mathematical context beyond textbook problems. Select survey results about students' attitudes towards the project are shared.

**Andrew Leahy** Knox College

### Geometrical Approaches to Calculus Problems

The claim that Newton and Leibniz invented the calculus, while technically correct, overlooks a rich 2000-year pre-history of calculus in which mathematicians pushed the language of Euclidean geometry to its limits in order to solve tangent, area, volume, and arc length problems. In this talk we will describe some of the original sources from Archimedes to Isaac Barrow that we have used to tell this history, some of our own translation. While these vignettes have been used primarily in history of mathematics classes, they are also appropriate for a geometry course targeted at pre-service teachers or, individually, to motivate topics in a calculus course.

**Meagan C. Herald** Viginia Military Institute

### From History of Mathematics to Topics Courses

Through teaching the history of mathematics course I realized the impact using primary sources can have with the students who would otherwise disengage in the course. I begin to branch out and use readings from primary sources in my linear algebra course and even created a 1-credit topics course based on primary sources with population and Markov Chain modeling. To focus the student's attention, I selected passages of pivotal articles assigned for homework and later discussed in class. The articles ranged from the early 1800's through the late 1900's and were well received by the students, with a few comments along the lines of "I never knew why we studied this until now."

**Carl Lienert** Fort Lewis College

### Understanding Desargues in an Historical Context

We'll discuss a student project from an historical perspective concerning the role of the work of Desargues in the development of Projective Geometry. What were the results he would have known, what did he contribute, and why is his most famous, but almost lost, result key in modern textbooks? Despite the warnings in many modern geometry textbooks, we'll attempt to read the primary sources.

**David Ruch** Metropolitan State University of Denver
**Joshua Gonzales** Metropolitan State University of Denver
**Ahern Nelson** Metropolitan State University of Denver

### Should we call it the Abel-Dirichlet theorem?

Primary source projects (PSPs) can be an effective teaching and learning tool for mathematics. This talk will discuss a PSP focused on infinite series for an undergraduate Real Analysis course. While trying to clean up a dodgy 1823 theorem by Cauchy on the continuity of infinite series functions, Abel stated and proved an 1826 theorem: if a power series $f(x)$ converges for $x = 1$, then $f(x)$ will converge to $f(1)$ as $x$ tends to 1. While this fact had been stated and used earlier, there seemed to be no rigorous proof published before Abel's. This result has become known as Abel's Theorem in many analysis texts. Unfortunately, Abel's proof was not entirely convincing to Liouville, who told Dirichlet of his concerns. Dirichlet then gave a significantly different proof that satisfied Liouville, and which he published after Dirichlet's death. Modernized versions of both proofs are given in various current texts. Our Real Analysis PSP includes elements of this story. Students who worked through this PSP in Spring 2018 will engage in a good natured debate regarding whose proof is better: Abel's or Dirichlet's?

**Zoë Misiewicz** SUNY Oneonta and ISAW-NYU

#### An Old Babylonian Procedure Text and a Table of Reciprocals

At the conclusion of a unit focused on primary sources for Old Babylonian mathematics, students have become familiar with the base-60 number system used on clay tablets almost 4000 years ago and have explored procedure texts that describe the geometric method of completing the square. Their final activity asks them to combine this knowledge with new information gleaned from an unfamiliar tablet in order to decipher and solve the following problem posed by an ancient scribe: "The *igibûm* beyond the *igûm*, 7 it goes beyond. *igûm* and *igibûm* what?" In addition to giving the relationship needed to solve this system of equations in two unknowns, the tablet provides practice in handling base-60 numbers with fractional parts and invites exploration of how prime factorization relates to terminating sexagesimal expressions. This activity is particularly valuable for pre-service elementary teachers, as students report gaining new appreciation for the challenges of a familiar topic seen in a new light.

## Part B: Saturday, August 4, 1:30–5:30 PM, Governor's Square 12, Plaza Building

**Matthew Haines** Augsburg University

#### An Activity on Letter Correspondence in the History of Mathematics

To build an appreciation of the communication of mathematics before email, students in a history of mathematics course are assigned to read portions of letters between mathematicians. After reading portions of a translation of the Pascal-Fermat Problem of Points correspondence and supporting documents, students are assigned to write original mathematics questions to each other, mail their letters, and subsequently reply to the letters they receive. This presentation reports on the student letters and their subsequent reflections of the activity.

**Edward Bonan-Hamada** Colorado Mesa University

#### Developing a *growth mindset* using TRIUMPHS PSPs

Psychologist Carol Dweck's idea of a *fixed/growth mindset* is an evidence-based model of attitudes people have about themselves which greatly influences how people learn and adapt. The application of this work to learning mathematics has been primarily addressed by Jo Boaler but widespread knowledge of *mindset* within the college-level mathematics community is somewhat limited. This talk will briefly introduce the idea of *mindset* and propose how using TRIUMPHS PSPs can help students develop a *growth mindset*.

**Janet Heine Barnett** Colorado State University - Pueblo
**Cihan Can** Florida State University
**Kathleen Clark** Florida State University

#### Beyond Just Doing the Math: an Investigation of the Role of Primary Source Projects in Supporting Student Learning of the Meta-discursive Rules of Mathematics

Recent research on the use of primary sources has suggested powerful connections with Sfard's theory of mathematics as a discourse. This theory defines learning as becoming a participant within a new discourse (versus a view of learning as the acquisition of knowledge), and discourse as an activity regulated by rules at two levels. *Object-level rules* reflect regularities in the behavior of the actual discursive (e.g., mathematical) objects - the rules involved in "just doing the math." *Meta-discursive rules* reflect the regular nature of the discourse itself. For example, it is the meta-discursive rules that determine what constitutes a proper definition, or whether a proof is correct and complete - precisely those aspects of mathematical practice that present students with the greatest challenges. This preliminary report describes an investigation of the potential for motivating students to "figure out" the meta-discursive rules of a new discourse by engaging with a specific type of Primary Source Project (PSP) being developed by the NSF-supported *TRIUMPHS* (*TRansforming Instruction in Undergraduate Mathematics via Primary Historical Sources*). We will illustrate Sfard's theory via an analysis of the meta-discursive rules present in a PSP based on excerpts from letters exchanged by Gaston Darboux (1843-1917) and Jules Houël (1823-1886) in which they debated issues related to rigor in analysis, and present samples of student data collected in an introductory analysis course in which this PSP was used.

**Kenneth M. Monks** Front Range Community College - Boulder County Campus

#### Implementing Primary Source Projects Using Overleaf, a Latex Platform in the Cloud

The TRIUMPHS NSF Grant team has created an incredible library of Primary Source Projects. However, many of these are above the level of first or second year undergraduates. Here I will demonstrate how I have implemented these projects in Overleaf, a cloud system for collaboration on LaTeX documents, allowing me to provide better support outside of class, allowing my students to help each other, and exposing them to a valuable tool along the way!

**Abe Edwards** Michigan State University

### Bridging the Sciences and the Humanities with Primary Historical Sources

British scientist C.P. Snow famously (and controversially) claimed that intellectual life in the late Twentieth Century was increasingly characterized by a split between two cultures - the sciences and the humanities. Snow saw a world in which many educated people were highly conversant within the bounds of their specific academic culture, but few were able to move fluidly across disciplinary boundaries. Recent decades have seen great changes in science, the humanities, and education. While significant divisions remain, academic life is increasingly interdisciplinary. In this talk, I discuss my use of primary historical sources in mathematics courses ranging from College Algebra through Calculus III. In particular, I describe how I use primary source readings as a powerful method for helping students bridge the "two cultures" . After several semesters of this work, we are starting to see students who view mathematics as a human endeavor, and who are starting to see themselves as human beings who do mathematics.

**Dan Kemp** South Dakota State University

### Supplementing the History of Mathematics with Original Sources

A traditional History of Mathematics course, based on Boyer's text, has been improved by including in the student's reading assignments, excerpts from original mathematical documents, in English translation of course. My students read excerpts from Euclid (of course!), Ptolemy, Diophantus, Tartaglia, Descartes, Wallis, Leibniz, Newton, Euler, Cauchy, Riemann, Dedekind, Cantor, and Andrew Wiles. This talk will describe how the excerpts are used, what the students do with them, and my expectations from these readings. My students are a mixture of pre service teachers (freshmen and sophomores), engineering majors seeking to complete a minor in mathematics (juniors and seniors), and a few interested mathematics majors. Due to the audience the mathematical level remains at elementary calculus. Nevertheless, reading from the masters can give the students insights as to where the mathematics they know comes from and they seem to find that very interesting.

**Qin Yang** MSU Denver

### Learning Mathematics through Historical Projects

I always try to use innovative approaches to my teaching. In particular, I have been using some Primary Source Projects (PSPs) in Calculus I, Introduction to Math- ematical Proofs and Discrete Mathematics for Computer Science. In September 2016, I attended a workshop about Transforming Instruction in Undergraduate Mathemat- ics via Primary Historical Sources (TRIUMPHS). TRIUMPHS is a National Science Foundation (NSF) funded project designed to develop, implement and disseminate historically-grounded projects in various branches of mathematics. As a TRIUMPHS site tester, I have used a mini primary source project (PSP) The derivatives of the sine and cosine functions, written by Dr. Dominic Klyve, in Calculus I class and a PSP Investigating Difference Equations, written by Dr. Dave Ruch, in Discrete Mathematics class. Many students are not only very interested in the primary source material included in these PSPs, but can understand concepts better and are more actively involved in their learning when working with PSPs. In this talk, I'll share my experience and that of my students using these two PSPs.

**Jerry M. Lodder** New Mexico State University

### The Radius of Curvature According to Christiaan Huygens

We examine the historical origin of curvature from Huygens's pioneering *The Pendulum Clock,* where he describes a clock that theoretically keeps perfect time in order to solve the longitude problem during the 1600s and 1700s. His description of the radius of curvature is entirely in terms of geometric segments, which he captures as the ratio of certain sides in similar triangles. This greatly illuminates the opaque definition of curvature found in calculus texts as the magnitude of the rate of change of the unit tangent vector with respect to arc length. Finally, as a student exercise we show how assigning differentials to Huygens's geometry results in the derivative formula for curvature (in terms of the first and second derivatives). The historical source adds context, motivation and direction to understanding the origin of curvature and its relation to an acceleration vector.

## The Capstone Experience for Mathematics Majors

### Saturday, August 4, 1:30–3:45 PM, Governor's Square 10, Plaza Building

**Organizers: Jacci White** Saint Leo University
**Monika Kiss** Saint Leo University
**Kevin Murphy** Saint Leo University

Capstone experiences vary from research, service, and artistic projects, to oral or written exams, to study abroad, internships, and more. Come and share your experiences and learn what others are doing with culminating experiences for Mathematics majors. We encourage the submission of scholarly work including but not limited to original research, innovative ideas, projects, curricular materials, assessment models, etc.

**Jacci White** Saint Leo University

### Contracts and Assessment of Senior projects

This session will examine the structure, pay, and documentation for senior projects. This experience includes the initial contract between the student and their faculty reader, the relationship between the University, course instructor, and the Faculty Reader, the grading rubrics used for the final project and presentation, as well as the criteria used to determine those who graduate with "Honors Program Distinction in Mathematics". Mathematics Majors at Saint Leo University are all required to complete a senior capstone project in mathematics, assessed by a primary course instructor as well as an individual faculty mentor, and presented before a board of mathematics professors. To maintain consistency in assessment between mathematics projects, contracts and rubrics were adapted from the Honors Program at Saint Leo University. After successfully implementing the process within the Mathematics Department, it is ready to share with the broader mathematics community.

**Leslie Jones** University of Tampa

### The Capstone Experience: An Individualized Approach

The Capstone Experience: An Individualized Approach We are a medium-sized comprehensive university with a small mathematics program. Our capstone course is a one-semester-hour independent study in which each graduating senior is paired with a faculty member. I will present the four most interesting senior seminar projects I have supervised over the past five years: the physics of billiards, sabermetrics for which the student completed original research, a stochastic model for waiting times at an office supply store, and a proof of Dilworth's Theorem. I will discuss issues of motivation, grading, and the often challenging time commitment.

**Ian Pierce** US Air Force Academy
**Beth Schaubroeck** US Air Force Academy

### Ten years of Math Capstone "for all" at the Air Force Academy

Over the last decade, math majors at the Air Force Academy have been required to complete a two-semester capstone project as a graduation requirement. This project culminates in the writing of an undergraduate thesis and typically also requires the student to give a presentation at a professional conference. We provide an overview of the course; to include variations in the classroom activities and supplemental assessments, the use of individual and group formats in project work, and the wide variety of sources for projects. We will also discuss the evolution of the course over time and the rationale behind particular decisions about our current Capstone model in the context of our overall curriculum and assessment practices.

**Doug Chatham** Morehead State University

### Is 2+1 better than 3? Examining Morehead State's two-semester senior capstone

As part of its general education program, Morehead State University (a regional comprehensive university located in rural northeastern Kentucky) requires its students to pass a three-hour capstone course in their major. In an attempt to give students more time to do their capstone project, in 2011 we split our department's capstone into a two-semester sequence in which each student works with a volunteer faculty advisor to conduct undergraduate research and then presents the results of their work through scholarly papers and public presentations. We discuss how well our two-semester sequence satisfies the expectations of the department and the MSU general education program.

**Michael Brilleslyper** U. S. Air Force Academy
**Ethan Berkove** Lafayette College

### The Long and Winding Research Project

Mathematics Majors at the Air Force Academy must complete an independent research project as a requirement for completing the major. Students work with a faculty advisor to learn something new, tackle a problem, and hopefully have a true "research experience." This talk is the story of one such project and the various lessons learned along the way. While every capstone project and every student is unique, there are some overarching features and methods that result in a better experience for both the student and the faculty member. Our student's research was in the area of graph labeling. As we describe the project, we will highlight some key observations we made along the way, including how to pick a topic to research and keeping it moving; how to set expectations; how to engage the student with the project and the broader mathematical community.

**Karen B. Stanish** Keene State College

### Investigate, Review, and Present: A Capstone Experience

The Capstone in Mathematics course at Keene State College has three main objectives: to investigate an area of mathematics that requires students to reason and problem solve at the senior level, to review and integrate previous learning in the mathematics major, and to produce significant a written report and an oral presentation on a mathematical topic. In this presentation, I will discuss the course design and assessment methods that I used to accomplish these objectives in the Capstone in Mathematics course. In particular, I will talk about the use of graph theory as a topic of investigation for the class, the assignment of review problems integrated into homework assignments, and the design of the major course project, which culminated in a written report and oral presentation. I will use evidence from student work, student reflections, and course evaluations to discuss both what worked in the course and what I will change when I teach the course again in the future.

**Kevin Murphy** Saint Leo University

### Writing Someone Else's Senior Exam

This talk will discuss some advantages to having a Senior Exam written by someone outside of the department as a means of truly assessing the skills developed by your program. Specifically, this talk will highlight an exam I wrote for another school's Senior Capstone Experience, and share some ideas for how to ensure students have a memorable conclusion to their undergraduate mathematics degree.

# General Contributed Poster Sessions

The MAA is pleased to announce the inaugural General Contributed Poster Session (GCPS) at MathFest 2018 in Denver. We will rotate the poster categories throughout the meeting and the number of rotations will depend on the number of accepted posters. The MAA will provide corkboards – you just need to bring your poster.

## General Contributed Poster Session I

### Thursday, August 2, 1:30–3:00 PM, Plaza Exhibit Hall

**Emil D. Schwab** The University of Texas at El Paso
**Gabriela Schwab** El Paso Community College

#### A tale of links between arithmetic and poset's Möbius functions

The classical Möbius function is an important arithmetic function in number theory. The poset's Möbius function was the subject of Rota's paper published in 1964 which laid the foundation of the modern combinatorics. We will investigate links that can be established between Möbius functions relative to arithmetic convolutions and poset's Möbius functions. These will be extended to special semigroups (as the bicyclic semigroup) and their corresponding posets. Finally, a category approach of the above is described using the concept of Möbius category in the sense of Leroux.

**Farhani Farhani** Bandung Institute of Technology
**Irawati Irawati** Bandung Institute of Technology

#### Annihilator ideal based zero divisor graph of $Z$ modulo $n$ over $Z$, complemented condition and girth

Annihilator ideal-based zero-divisor graph, denoted by $\Gamma_{Ann(M)}(R)$, is the graph whose vertices are $x \in R \setminus Ann(M)$ but $xy \in Ann(M)$ for $y \in R \setminus Ann(M)$; and two distinct vertices $x$ and $y$ are adjacent if only if $xy \in Ann(M)$. A vertex $x$ in graph $G$ is called a complement of $y$ in $G$, if $y$ adjacent to $x$ and no vertex adjacent to both $x$ and $y$. A graph $G$ is called complemented if every vertex of $G$ has complement. Jesse Gerald Smith Jr already determined complemented condition for commutative ring using radical properties and quotient ring , but this article is about finding complemented condition of Annihilator of Module $Z_n$ over $Z$ using definition and prime number property and also include the girth of the graph. Eventually, $\Gamma_{Ann(Z_n)}(Z)$ complemented if and only if $n = p_1p_2 ::: p_k; p_i$ different prime; and the girth of $\Gamma_{Ann(Z_n)}(Z)$ is either 3 or 4, equal to 4 if and only if $n = p_1p_2$ for $p_1, p_2$ different prime.

**John E. Foster** Walla Walla University

#### Computer-Assisted Calculation in Hopf Algebra Representations

Often it is helpful to understand a particular example before attacking a general problem. Unfortunately, sometimes even basic examples can involve very tedious computations. The author found this to be the case when investigating representations of the quantum double $D(U_q(\mathfrak{sl}_n))$ even for $n = 2, 3$. We will see how a Smalltalk computer program, together with SageMath, can calculate actions of this Hopf algebra and guide our search for a good set of generators and relations in its algebra of finite-dimensional representations.

**Saul Blanco** Indiana University
**Charles Buehrle** Notre Dame of Maryland University

#### Some relations on prefix reversal generators of the symmetric and hyperoctahedral group

The pancake problem is concerned with sorting a permutation (a stack of pancakes of different diameter) using only prefix reversals (spatula flips). Although the problem description belies simplicity, an exact formula for the maximum number of flips needed to sort $n$ pancakes has been elusive. In this paper we present a different approach to the pancake problem, as a word problem on the symmetric group and hyperoctahedral group. Pancake flips are considered as generators and we study the relations satisfied by them. We completely describe the order of the product of any two of these generators, and provide some partial results on the order of the product of any three generators. Connections to the pancake graph for the hyperoctahedral group are also drawn.

**Thomas Cameron** Davidson College

### A Practical Parallelizable Fourth-Order Modification of Laguerre's Method

We present a novel modification of Laguerre's method that has strong virtues including local fourth-order convergence to simple roots and a workload that is embarrassingly parallel. This modification results in an algorithm for the concurrent approximation of all roots of a univariate polynomial. We present the results of a detailed analysis of our algorithm along with several numerical experiments that verify our analysis and demonstrate the effectiveness of our Fortran 90 implementation in comparison with other open source polynomial root solvers. Finally, we present the progress of a current student project whose goal is to use multi-precision and multi-threading techniques to improve the speed and accuracy of our algorithm.

**William Feldman** University of Arkansas

### Comparisons of locally determined nonlinear maps and generalized orthomorphisms

For $C(X)$, the Banach lattice of all continuous real-valued functions on a compact topological space $X$, an orthogonally additive (not necessarily linear) map $T$ from $C(X)$ to itself is said to be locally determined if $Tf(x) = Tg(x)$ whenever $f = g$ on a neighborhood of some fixed point $x^\wedge \in X$. $T$ is said to be an orthomorphism if $f$ orthogonal to $g$ implies $Tf$ is orthogonal to $g$. These properties are related and consequences for more general Banach lattices are considered. The results are incorporated into characterizations of orthogonally additive maps as linear maps composed with orthomorphisms.

**Hudson Akewe** University of Lagos

### Convergence speed of some random implicit-Kirk-type iterations for contractive-type random operators

The main aim of this paper is to introduce a random version of multistep type iterative scheme called a modified random implicit-Kirk multistep iterative scheme and prove strong convergence and stability results for a class of generalized contractive-type random operators. The rate of convergence of the random iterative schemes are also examined through example. The results show that our new random implicit kirk multistep scheme perform better than other implicit iterative schemes in terms of convergence and thus have good potentials for further applications in equilibrium problems in computer science, physics and economics.

**Russell W. Howell** Westmont College

### Do Annular Functions Abound?

Annular functions are complex-valued functions analytic in the open unit disk $D$ with the property that their minimum modulus converges to infinity on a series of Jordan Curves that converge outward to the boundary of $D$. Many years ago F. Bagemihl and P. Erdos investigated the zero distribution of these functions, whose construction involved power series with large gaps. A surprising result is that such functions occur as representatives of the simplest of all power series, viz., ones whose coefficients are plus or minus one. By associating sequences of these coefficients with zeros or ones as a binary decimal gives a real number in the interval [0,1]. It is known that the set of such numbers forming an annular function is a residual set. The zero-one law then dictates that its measure is either zero or one. What is it?

**Aaron Melman** Santa Clara University

### Generalizations of the Enestrom-Kakeya theorem

The Enestrom-Kakeya theorem is an elegant theorem that establishes upper (and lower) bounds on the moduli of zeros of polynomials with positive coefficients. We show how it can be generalized and improved in a unifying way by relying on just two tools: appropriate polynomial multipliers and the generalization of an observation by Cauchy. In this way, we derive zero inclusion regions composed of one, two, three, or more smaller disks, rather than just the one disk obtained by the Enestrom-Kakeya theorem.

**Anna Seitz** Iowa State University
**Mary Vaughan** Iowa State University
**Nate Harding** Iowa State University
**Emelie Curl** Iowa State University

### Vector Reconstruction: A Generalized Kaczmarz Algorithm

The Kaczmarz Algorithm is a method for reconstructing vectors in a Hilbert Space, $\mathcal{H}$. Using inner products with a so-called "effective sequence," the Kaczmarz Algorithm can be applied at any vector $x \in \mathcal{H}$ to generate a sequence of approximations which converges to $x$ in norm. Although a useful class of sequences, effective sequences are generally intolerant towards perturbation. To obtain more flexibility, we explore the idea of "effective pairs," two sequences that work together to analyze and synthesize in a generalized Kaczmarz Algorithm. After the pattern of effective sequences, we seek a complete characterization of these pairs via a partial isometry condition on an associated operator, $V$.

**Junyuan Lin** Tufts University

### An AMG Approach in Solving Graph Laplacians of Protein Networks Based on Diffusion State Distance Metrics

In this presentation, protein networks from 2016 Disease Module Identification DREAM Challenge are analyzed. We redefined the Protein-Protein Interaction networks on a new distance metric, "Diffusion State Distance" metric, and applied a modified Algebraic Multi-grid Method to calculate the distance between each pair of nodes. Finally, we applied spectral clustering to partition the protein network into functional modules. Consequently, we ranked No.1 out of all competing teams over the world.

**Tung Thai** Wentworth Institute of Technology

### A Cost Benefit Analysis of Cyber Defense Improvements

This project focuses on analysis and reduces the losses caused by cyber-attacks on a power grid. In the past few years, several major cybersecurity attacks on supervisory control and data acquisition (SCADA) devices have been reported. Such attacks can result in damages to the economy and have an impact on society. Existing research has been concentrated on determining vulnerability assessment of SCADA systems and investigating the effect of improved countermeasures. The model presented is designed to predict the losses for a variety of attack scenarios and identify the most effective way to improve the system. Monte Carlo simulation is used to perform cost-benefit analyses concerning proposed system improvements. The process is completed according to the following steps: 1. Identify the intrusion scenarios based on a threat tree. 2. For each successful attack, simulate the losses utilizing an appropriate distribution (generalize logistic function aka Richard's curve) 3. Simulate ten years of attacks in a manner consistent with the adjusted vulnerabilities to find the losses of each intrusion scenario. 4. Compare simulated losses over a variety of improvement schemes to advice the prioritization of system improvements.

**Laurentiu Sega** Augusta University

### Reduced fertility and asymptotics of the logistic model

This logistic model includes three age groups. Juveniles do not reproduce, and old individuals reproduce at a reduced rate. Pairings between individuals of different fertility rates may lead to multiple equilibria and bi-stability: the total population converges to different limits depending on its initial size. The behavior is correlated with transition rates from high to low fertility groups and with the frequency of pairing among the various groups of reproduction level. The proportions of adults at equilibrium are roots of a quartic polynomial, alternating sinks and saddles. Necessary and sufficient conditions for the existence of bi-stability are provided for a simplified model.

**Karen L. Collins** Wesleyan University
**Megan E. Heenehan** Eastern Connecticut State University
**Jessica McDonald** Auburn University

### Clique Immersion in Graph Products

Let $G$ and $H$ be graphs and $G * H$ represent a particular graph product of $G$ and $H$. We define $im(G)$ to be the largest $t$ such that $G$ has a $K_t$-immersion and ask: given $im(G) = t$ and $im(H) = r$, how large is $im(G * H)$? Best possible lower bounds are provided when $*$ is the cartesian or lexicographic product, and a conjecture is offered for each of the direct and strong products, along with some partial results.

**Liz Lane-Harvard** University of Central Oklahoma

### Iterated Line Graphs of Trees and Bi-Regular Graphs

In 1965, van Rooij and Wilf considered sequences of line graphs, in which they grouped sequences of line graphs into four categories. To this, we'll present results on sequences of line graphs for star graphs. We will then investigate slight variations of star graphs. Utilizing these results, we'll consider the line graphs of bi-regular graphs.

**Joshua Steier** Seton Hall University
**Kristi Luttrell** Seton Hall University
**John T. Saccoman** Seton Hall University

### Limit characterizations through spanning trees in multigraphs: an exploration

Nikolopolous et. al., using Cayley's Theorem and Kirchhoff's Matrix Theorem, established limiting results for the number of spanning trees of certain families of graphs. Focusing as they did on edge deletions, we applied similar techniques to similar families of multigraphs. We examined existing results involving threshold graphs and split graphs. Utilizing various matrix properties and a general formula for the number of spanning trees on complete multigraphs, we conjecture limit characterization for multigraphs with arbitrary multiplicity. Keywords: multigraphs, split graphs, spanning trees, Laplacian Matrix, Kirchhoff's Theorem

**Espen Slettnes** University of California, Berkeley

### Minimal Embedding Dimensions of Rectangle *k*-Visibility Graphs

Research on bar visibility graphs was originally motivated by problems about constructing VLSI (Very Large Scale Integration) circuits, and were adopted in the 1980s as a geometric model to represent traces, e.g. on circuit boards and in VLSI chip designs. Rectangle visibility graphs were introduced by Bose et al in 1997 as a generalization of bar visibility graphs. A graph is a rectangle visibility graph if it can be represented with vertices as disjoint axis-parallel rectangles, such that there is an unobstructed axis-parallel line of sight between two rectangles if and only if there is an edge between the corresponding vertices. We combine rectangle visibility graphs with $k$-visibility to form rectangle $k$-visibility graphs, in which the line of sight between two rectangles in the representation can be obstructed by at most $k$ other rectangles. We then take a natural generalization of rectangle $k$-visibility graphs into higher dimensions. We find that given enough spacial dimensions there exists a rectangle $k$-visibility representation of any graph $G$, and find upper bounds on the smallest such dimension needed to represent various graphs. We continue to study its properties, and proceed to bound it for complete graphs, complete $r$-partite graphs, and hypercube graphs.

**Joel Salazar** California State University, San Bernardino

### Radio Number for Ninth Power Paths

Let $G$ be a connected graph. The *distance* between two vertices $u$ and $v$ in $G$ is defined by the length of the shortest path in $G$ between $u$ and $v$, which is denoted by $d(u, v)$. The *diameter* of $G$, denoted $\text{diam}(G)$, is the maximum distance between two vertices in $G$. A *radio labeling* of $G$ is a function $f$ that assigns each vertex a non-negative integer such that $|f(u) - f(v)| \geq \text{diam}(G) - d(u.v) + 1$ holds for any two distinct vertices $u$ and $v$ of $G$. The *span* of $f$ is the difference between the smallest and largest channels used. The *radio number* of $G$, denoted $\text{rn}(G)$, is defined as the minimum span of all radio labelings of $G$. $f$ is said to be an *optimal radio labeling* if $span\ f = \text{rn}(G)$. A *path* on $n$ vertices, denoted $P_n$, is a connected graph whose vertices can be ordered so that two vertices are adjacent if they are consecutive in the order. A ninth power path, $P_n^9$, is obtained from $P_n$ by adding edges between any two vertices whose distance is less than or equal to nine. We will provide a lower bound for the radio number of $P_n^9$ and prove a method for producing the minimal span of our radio labeling.

**Taylor Short** Grand Valley State University

### The saturation number of single-defect carbon nanocones

The saturation number of a graph is the cardinality of a smallest maximal matching. The problem of determining the saturation number is related to edge dominating sets and efficient edge dominating sets in a graph. The saturation number also has implications in chemistry regarding the adsorption of dimers to a molecule. This presentation concerns nanocones, which are a type of carbon network situated between graphene and fullerene in terms of structure. We present asymptotically tight upper and lower bounds for the saturation number of single-defect carbon nanocones.

**Hector Dondiego** Lewis University
**Chandler Stimpert** Lewis University

### Using Graph Theory to Design Optimal Strategies for DNA Self-Assembly

Motivated by the recent advancements in nanotechnology and the discovery of new laboratory techniques using the Watson-Crick complementary properties of DNA strands, formal graph theory has recently become useful in the study of self-assembling DNA complexes. Construction methods developed with concepts from undergraduate level graph theory have resulted in significantly increased efficiency. In this poster, we present the results of a summer undergraduate research project applying graph theoretical and linear algebra techniques to constructing complexes like general bi-partite graphs and lattice-based graphs which can be created from self-assembling DNA. In particular, we explore various design strategies given three different laboratory constraints.

**Matthew Hudelson** Washington State University

### Enumerating Multiple Frog Paths

We consider enumerating collections of non-intersecting N/E paths (paths on the integer plane consisting of (1,0)- or (0,1)-steps) that begin at given points and end at given points. We show that the number of such collections can be written as a determinant of a matrix whose entries are binomial coefficients, each entry constructed from information about the number of paths, the starting points, and the ending points.

**Cyrus F. Nourani** Acdmkrd AI Berlin
**TBA TBA** Acdmkrd AI Berlin

### A Brief on Direct Product Models and Languages

A Brief on Direct Product Languages and Models Languages $L1, ..., L\omega$ with signatures $\Sigma i$'s, ..., $\Sigma i < \Sigma i + 1$ Apllying inclusion ordering on the sigantures $\Sigma *$'s we have morphic preorders on the $T_{\Sigma} i$. Proposition There is a small complete category on the infinitary language fragments definable with the $\Sigma i$'s based on the direct product on $T\Sigma i$'s Theorem There is a generic functor on the category the omitting n-types realizing a direct product model. Proof.is based on having the small complete category with a filtered limit on monads to the n-types.

**Joshua Boone** Lincoln Memorial University

### Matrix Powers and Symmetric Polynomials

Armed with only the Cayley-Hamilton theorem and mathematical induction, we write a formula for $A^k$ for any $n \times n$ matrix $A$ with $k \geq n$. This formula depends on $A,\ A^2,\ \ldots\ ,\ A^{n-1}$ and the symmetric polynomials $h$ and $e$ evaluated at the eigenvalues of $A$. Noticing a clever recursion, we evaluate these polynomials without explicitly finding the eigenvalues.

**John C. Wierman** Johns Hopkins University

### Upper Bounds for the Bond Percolation Thresholds of the Cubic, Body-Centered Cubic, and Face-Centered Cubic Lattices by a Growth Process Approach

We establish upper bounds for the bond percolation thresholds of the simple cubic, body-centered cubic, and face-centered cubic lattices. Each bound is obtained using a three-dimensional growth process which explores the open cluster as a dynamic process, expanding only within a restricted set of edges at each step. The three-dimensional growth process is projected on a selected plane to obtain a two-dimensional growth process on a planar lattice. Stochastic ordering and coupling are used to relate the two-dimensional growth process to the bond percolation model on the planar lattice to obtain an upper bound.

**Jessica Oehrlein** Columbia University
**Gabriel Chiodo** Columbia University
**Lorenzo M. Polvani** Columbia University
**John Fyfe** Environment Canada
**Anne K. Smith** National Center for Atmospheric Research

### Effect of Solar Variability on North Atlantic Climate

The North Atlantic Oscillation (NAO) is the first principal component of the atmospheric system in the North Atlantic region. The NAO has been suggested to respond to the 11-year cycle in solar activity with a lag of a few years. The solar/NAO relationship provides a potential pathway for solar activity to modulate surface climate. We investigate the robustness of the solar/NAO signal in four observational data sets and in long integrations of a climate model forced with the 11-year solar cycle. Using multiple linear regression, we confirm the existence of an apparent solar/NAO signal across the observational data sets and accurately reproduce many features of the observed signal in our model simulations. However, in both the observations and model simulations, continuous wavelet transforms show that the signal is non-stationary. Moreover, we find that an apparent NAO/solar signal can also be detected in model integrations with no 11-year solar cycle forcing. This suggests that the correlation found in observational data is the result of internal decadal variability in the NAO and not a response to the solar cycle.

**Ethan Berkove** Lafayette College
**Michael Brilleslyper** U. S. Air Force Academy

### Fibonacci Identities: No Induction Required

The Fibonacci numbers are among the most well-studied sequences in mathematics. There is a vast literature dealing with Fibonacci identities, ranging from simple sum formulas to extremely complex expressions. Several of the more well-known identities involve sums of Fibonacci numbers and their proofs frequently occur as textbook problems in mathematical induction. Here, we present a construction using polynomial long division that results in a simple formula that can be used to generate an infinite collection of partial sum-type Fibonacci identities, some well-known and others much less so. The result completely bypasses the need for mathematical induction. The underlying theory of the method comes from the general and well-known framework of generating functions for linear recurrence relations, so our results are easily modified to other sequences, like the Lucas numbers and the Tribonacci numbers.

**David Terr** UC Berkeley

### Generalized Zeckendorf Expansions

In this paper we show how to generalize the Zeckendorf expansion, which is the unique expression of an arbitrary nonnegative integer as the sum of distinct nonconsecutive Fibonacci numbers, to another unique expression involving integer multiples of elements of a sequence, which we call a base sequence, defined by an arbitrary real number $\gamma > 1$. The familiar Zeckendorf expansion is a special case with $\gamma = \alpha$ and for an arbitrary integer $b \geq 2$, the base-$b$ expansion is a special case with $\gamma = b$ and base sequence consisting of the nonnegative integer powers of $b$. We also show how to add and subtract arbitrary positive integers using their generalized Zeckendorf expansions.

**Kryssa C. Goodhart** Rowan University
**Jay L. Schiffman** Rowan University

#### Primes and Divisibility Patterns in the Repdigit Sequence 3, 31, 311, 3111, 31111,...

The modified repdigit (repeated digit) sequence consisting of the digit three followed by all one's serves as an appealing integer sequence. In this poster, the authors demonstrate using modular arithmetic recurring divisibility patterns as well as when early primes initially enter the sequence. In addition, we exhibit the complete factorizations for the initial eighty terms in the sequence as well as determine which primes $<$ 500 never enter the sequence as factors.

**Aubrey R. Laskowski** University of Illinois at Urbana-Champaign
**Michael J. Schirle** University of Illinois at Urbana-Champaign

#### Recent Developments on Stern's Diatomic Sequence and a Sister Function

Stern's Diatomic sequence (OEIS A002487) is a curious mathematical object but has enjoyed little exposure in mainstream mathematics. The sequence has connections to combinatorial number theory via enumeration of the rationals and the Minkowski ?-function. We will be presenting results on the maps $\sigma_k = \frac{s(n)}{s(n+k)}$ for fixed $k$, specifically on representing positive rationals in these forms. We also introduce a sister function of the Stern sequence which has a similar recurrence. This function provides exact formulas for previously unsimplifiable equations relating to the Stern sequence. In the effort to understand maps, we formulate continuous versions of the Stern sequence (and its sister) using analytic methods. Finally, we will discuss conjectures around the maps such as surjectivity and solutions to $\frac{s(n)}{s(n+k)} = c$.

**Robert Styer** Villanova University
**Reese Scott** Somerville, MA

#### Solutions to the Diophantine equation $X + Y = c^z$ when $XY$ is divisible by a fixed set of two primes

The Diophantine equation $X + Y = c^z$ when $XY$ is divisible by a fixed set of two primes. Consider $N$, the number of solutions in positive integers $(X, Y, z)$ to the equation $X + Y = c^z$, where $c$ is a fixed positive odd integer, and $XY$ is divisible by every prime in a set of $n$ primes and by no other primes, with $\gcd(c, XY) = 1$. Previously, we showed the following: letting $D$ be the least positive integer such that $(XY/D)^{1/2} \in \mathbb{Z}$, and letting $\omega$ be the number of distinct primes dividing $c$, standard elementary approaches use a bound of $2^n$ on the number of possible $D$ and a bound of $2^{\omega-1}$ on the number of ideal factorizations of $c$ in the field $\mathbb{Q}(\sqrt{D})$ which can correspond to a solution in which $(XY/D)^{1/2} \in \mathbb{Z}$, and obtain $N \leq 2^{n+\omega-1}$. We improved this by finding an inverse proportionality relationship between the number of $D$ which occur in solutions and a bound (independent of $D$) on the number of ideal factorizations of $c$ which can correspond to solutions for a given $D$. We obtain $N \leq 2^{n-1} + 1$. In this presentation we will consider the specific case when $n = 2$: let $\{d_1, d_2\}$ be the primes dividing $XY$; if $n = 2$ then $N \leq 2$, except for the following choices of $(d_1, d_2, c)$, taking $d_1 > d_2$: $(3, 2, 5)$, $(5, 2, 3)$, $(2^{g-1} - 1, 2, 2^g - 1)$. When $N = 2$, we will present several infinite families of solutions and the known anomalous solutions.

**Shigeru Masuda** RIMS, Kyoto University

#### The modeling and calculation of rise and fall of the liquid in capillary action by Poisson.

We discuss the problem of capillary action by Poisson (1831) from the history of mathematical physics, or, the modeling and calculation of the rise/fall of the liquid in the neighborhood of wall. He supposes the mutual action of attraction between the molecules, $\rho^2\phi(r)\omega\omega' dsds'$ with the function $\phi(r)$ of distance $r$ between two molecules. He separates the domain of the liquid into four parts $C$, $C'$, $D$ and $D'$, of which the two are near the wall: $C$ locates over $C'$, and other two, in the liquid, $D$ locates over $D'$. He seeks the unknown $\Delta$ from the $2R' - R \equiv \Delta$, where, $R$ and $R'$ the actions from the liquid and from the wall. Under the condition of constant density of liquid, $z + z'R = \rho^2\phi(r)r(1 - ku)(1 + k'u')dzdz'dudu'dsds'$, in putting and putting $k = k' = 1$, $z + z'q \equiv 2\rho^2\phi(r)r$ where, $q$ and $q$ the quantities, $\rho$ and $\rho$ densities of two material, $dz$, $dz'$, $du$, $du'$, $dx$, each elements of the distances. Using $c$ the contour and $R = qds = cq$, and integrating the function, he calculates the quantities of action $Q$, $Q'$, $P$ defined $Q$ in $D$, $Q'$ in $D'$ and $P$ in $C'$, under the condition of equilibrium $Q + Q' + P = 0$ in $D$, and, he gets $P = -2cq$, $Q' = R = cq$, $Q = \Delta = cq$. (On the $Q$, he shows another direct method.) By his hypothesis, it turns finally $q = q'$, $\rho = \rho'$, because of the constant density, namely, it means that the materials are equal between the tube and liquid. From this contradiction, he concludes the rise/fall dues to the abrupt change of variation in density of liquid near the wall. Our presentation invite the undergraduate to interest the interdisciplinary arena of mathematics (in particular, integral), physics and mathematical history.

# General Contributed Poster Session II

## Thursday, August 2, 3:30–5:00 PM, Plaza Exhibit Hall

**James Olsen** Western Illinois University

### Standards Based Assessment: An Easy-to-Use SBA Quiz System

In this session I will explain my SBA (Standards Based Assessment) Quiz system (most of which I adapted from another teacher). I have found it to be a great way to do formative assessment and increase student mastery of material. I will describe the features of this system which make it effective and easy to use –and how it has raised my students' understanding & scores on exams.

**David Thomas** University of Providence
**Ciaran Mac an Bhaird** Maynooth University, Ireland

### Mathematics Learning Support at US Colleges & Universities

In this talk, we will present preliminary findings from our Survey of Math Center Provision in US Higher Education. The survey will provide a first systematic look at mathematics learning support (MLS) in Math Centers across the country, and allow for a comparison with findings in similar studies conducted in Ireland, the UK and Australia. It will also provide a point of departure for Math Center faculty and staff interested in the improvement of MLS. For decades, US Higher Education has struggled to meet the expectations of parents, students, and political leaders who believe that essentially all high school graduates deserve a college education. Unfortunately, many high school graduates are unprepared for college level mathematics. For instance, more than 60% of students entering 2-year institutions are required to pass at least one developmental math course before enrolling in college-level math courses. Similar challenges face students entering 4-year colleges and universities. Nationwide, publicly funded institutions are reducing budgets for classroom-based developmental mathematics programs. As a result, Math Centers are becoming increasingly important spaces for addressing student retention and transition issues.

**Jonathan Hulgan** Oxford College of Emory University

### A Comparison of Two Approaches to Teaching Calculus I

Active learning techniques have been a common focus of recent pedagogical reforms in mathematics. Such approaches focus on having students actively engaged in some sort of determined action during class rather than merely listening and recording notes. The degree to which students are required to actively participate in class varies from technique to technique; seemingly little research exists in the way of pinpointing where the ideal balance between student activity/practice and direct instruction in the classroom lies—or if such a balance exists. To this end, two active-learning approaches were applied to a first-semester calculus course at Oxford College. This work compares student performance and attitude between the two approaches and discusses inferred and observed pedagogical advantages of each approach in this course.

**Chuck Lindsey** Florida Gulf Coast University

### The "Lost" Books of Euclid's *Elements*

This talk will give an historical overview of the appearance, transmission, and (sort of) disappearance of Books XIV and XV of Euclid's *Elements*, through the various versions and editions that have been produced in several languages. Although these two Books are certainly not due to Euclid, for many centuries they were included as part of the standard text. The changing content and role of these Books track the evolving approaches to number and magnitude, and illustrate the progression of the *Elements* from core textbook to historical artifact.

**Wen-Haw Chen** Tunghai University

### A College Geometry Course Involving Civilization, Logic and Beauty

Geometry is closely related to culture, art, architecture, and the development of science and technology, as well as the exploration of the universe. The contents of geometry are suitable for general and liberal education curricula in higher education. These included understanding of the relationship between geometry and the development of civilization, and of rigorous logic in geometry, as well as the objective to experience the beauty of geometry. This study conducted action research in an innovative college geometry course to investigate the effect of the course design. The idea of course design is to blur the boundaries between general and professional courses and takes the history and applications of geometry as the core with interdisciplinary cooperative learning system. Here cooperative learning is facilitated by pairing major and non-major math students to discuss specified geometric topics from each other's professional perspectives. The result of this study shows positive impacts on the learning outcomes of students.

**Jamie Sutherland** University of Delaware

### Opening Up the Transitions Course: New Proof Tasks for the Creative Math Major

Does teaching mathematical proof to future teachers leave you feeling more like a drill sergeant than an inspiring artist? Do students groan in dismay instead of gasp in surprise when they encounter a novel new proof technique? Are the proofs your students hand in to you made of pre-cut, solid blocks of reasoning instead of unique expressions of the author's individuality? While there will always be a need for drill and practice in any kind of learning, a "transition to higher math" or "introduction to proof" class is a wonderful opportunity to introduce students to more open-ended problems and tasks than they have seen before in math class. Most typical math textbook exercises are predominantly closed problems. Students are expected to find specific solutions using particular procedures they need to perform in order to get full credit. Students very rarely see an open task that expects them to decide what an acceptable solution strategy would be. In a proof class, students are given the definitions, axioms, even statements that need to be proven or disproven. Opening up these problems allows students to play with the ideas. They can experiment and experience the learning that comes with failure without fear of not meeting some preset expectations. Come learn about open-ended tasks at varying levels of openness from defining to conjecturing to even "playing" with axiom systems. This presentation can appeal to both teacher educators and teachers of a "transition to higher mathematics" course. Examples from such a course will be given with variations that change how "open" the task is. Bring your ideas about what the goal of such a course should be and whether that goal should shift depending on the make-up of the students.

**Nicholas Long** Stephen F. Austin State University
**Jeremy Becnel** Stephen F. Austin State University

### Calculus in Virtual Reality

Often the largest hindrances to student success in multivariable calculus courses are the student's inability to visualize the curves, surfaces, and vector fields and this disconnect that this causes between the geometric interpretation and the algebraic calculation. While there are many great tools that are freely available (like CalcPlot3D) to help students understand these multivariable objects, the rendering is still a two-dimensional picture of a three-dimensional object. In order to show these objects with visual depth, we have created a virtual reality app that is available for free on smartphones and requires less than $5 in additional hardware costs for students. We have created interactive lessons and demos for multivariable calculus topics that allow the user to see visual depth as it would look in a three-dimensional setting. We have also integrated inexpensive Bluetooth controllers and the ability for the user to input their own expressions and explore the related figures. We will have several demonstration setups available for faculty to experience this immersive virtual environment.

**Yanping Ma** Loyola Marymount University

### Application Projects to Students in Calculus for Life Science

In order to study students' attitude towards math in calculus for life sciences courses, several application projects/problems have been developed from scratch to demonstrate the importance of math. The problems include discussions about Mean Arterial Pressure, Muscle physics, Schock Index and Modified Shock index in triage measures, Glycemic index and Michaelis - Menten Kinetics. In this poster, we will show some demonstrations of those projects, and report on students comments about the Michaelis-Menten Kinetics.

**Jeffrey M. Ford** Gustavus Adolphus College

### Blending Team-based Learning with Standards-Based Grading in a Calculus I classroom

In the Fall of 2017, two sections of Calculus 1 were taught using a Team-based learning model. Individual student assessments were graded using a set of 32 standard learning objectives. The blending of TBL with standards-based grading, caused many of the students to report a decrease in mathematics anxiety. Failures were seen as learning opportunities, rather than dead-ends. We present here a description of the method, the rate at which students met standards throughout the semester, and feedback from students on the efficacy of the method.

**James M. Talamo** The Ohio State University

### Developing Tightly Coordinated Calculus Courses for STEM Majors

In Autumn of 2015, the mathematics department received funding from the university to redesign its first and second semester calculus courses. Both of these courses are tightly coordinated; all sections of the course have a common calendar and syllabus, all students take common quizzes and exams, complete the same outside homework assignments, and all final grades are determined by a common grading scale. At the time, the Calculus II course was the course in the department with the lowest success rate. Over the following two years, the course was completely redesigned with a single goal in mind: to create a cohesive course environment in which students and instructors had a clear idea of departmental expectations by scaling up the individual classroom experience. This required a complete overhaul of the course resources, redesigned focused assessment material, the introduction of just-in-time review assignments, improved communication, and the pedagogical implementation of these changes and has led to improved student perception of the courses and significantly lower failure rates without changing the level at which the course material is assessed.

**Kristen Mazur** Elon University
**Laura Taylor** Elon University

### Student Perceptions of Engagement in Calculus 1

According to the MAA's Instructional Practices Guide mathematics instructors must work to engage students in the learning experience, since deep learning requires "student engagement with content both inside and outside the classroom." Many of us that teach mathematics have our own ideas on how to engage students in the classroom. But, what does "engagement" or "engaged learning" mean to our students? What types of activities do students consider to be engaging in the classroom? Moreover, do they agree with us, as their instructors, on what makes a particular lesson engaging? In this project, we asked students in Calculus 1 at Elon University to reflect on their perceptions of engagement in calculus. Both students and instructor rated the engagement level of each class throughout the semester. Students also completed an end-of-semester survey in which they listed the classroom practices that engaged them during the semester and gave their own definitions of engaged learning. We found that our students' perceptions and definitions of engagement vary greatly and, at times, disagree with what we, as instructors, define as engaging.

**Dan Kemp** South Dakota State University

### Using History to Motivate Calculus

Students come to calculus classes not knowing much about the history of mathematics. They seem to think that mathematics, and calculus in particular, is just another hurdle for them to jump over. Having student read a 'popular' history of mathematics while taking calculus gives them a new perspective about the subject and seems to positively affect their attitude about mathematics. I have had my students read a variety of mathematics history books and the anecdotal results will be described on this poster. After each reading assignment there is a brief discussion in class and then the students write a reaction (reflection) paper about the reading. This is where I find out what they really think. Most admit they dreaded the assignment, but everyone, yes everyone, had a positive reaction to reading about the history of mathematics. Papers were graded mostly for completion.

**Gizem Karaali** Pomona College

### Whose Math and For What Purpose? A Community Seminar on Identity, Culture, and Mathematics

In the spring of 2018, the Pomona College mathematics department hosted a community seminar on identity, culture, and power in the discipline and education of mathematics, titled "Whose Math and For What Purpose?" . The seminar was open to all students, faculty, and staff of the college as well as any members of the local community. In this poster I describe the specifics of the seminar, what types of issues we have discussed, what a typical seminar session looked like, and what we all were hoping to gain from the experience. Finally I summarize our semester-long experience and share ideas for the future. This seminar was facilitated by funds from the Consortium on High Achievement and Success (CHAS).

**Kyle Riley** South Dakota School of Mines & Technology

### Case Study of Student Success

The South Dakota School of Mines and Technology (SDSM&T) is a university focused on preparing students in engineering and science. All majors on the SDSM&T campus require students to pass at least Calculus 1 and the clear majority of students are in majors that require the entire calculus sequence plus Differential Equations. We analyze a case study concerning students from the introductory courses of College Algebra and Trigonometry from the fall semesters of 2015 and 2016. We use this small case study to investigate the success these students have in subsequent courses to reveal how these courses prepare our students to enter Calculus.

**Janet St. Clair** Alabama State University

### Beyond the Textbook - Stories, Cartoons, and More

Mathematics is dynamic and grows and changes over time. Because it is not collection of skills and facts, it is impossible to fully capture its nature in a "school" textbook. To encourage students in a finite mathematics class to develop a well-rounded view of mathematics and a deeper understanding of certain topics, they read stories related to mathematics, read presenter-created cartoons that gave them a sense of the history of certain mathematical topics and their connections to the real world, and investigated topics more deeply than presented in their textbook. To demonstrate use of stories, the presentation includes ideas/activities related to students' reading of the story, *The Tutor* by *Anton Chekhov*, done during the study of systems of linear equations. A presenter-created cartoon related to systems of linear equations in ancient China and Gauss' astronomy work is used to show not only how students developed a deeper understanding of topics but also how the presenter was able to better understand students' misunderstandings. Students' activities with the idea of representing sets other than using the typical Venn diagrams with rectangles and circles are used to further demonstrate how topics were investigated more deeply.

**Rachael Talbert** Towson University
**Diana Cheng** Towson University

#### Choreographing in Problem Solving: Mathematical Interpretations of Figure Skaters' Blade Tracings

In a Spring 2018 mathematics course on Problem Solving for Middle School Teachers, we implemented a model-eliciting activity related to the Olympic sport of figure skating. Pre-service and in-service teachers were asked to choreograph and draw patterns on an aerial view of a skating rink. Using expressions and equations to model the lengths of composite figures, students found the lengths of their designed paths. We show student work on this MEA and how it follows the steps of Zalman Usiskin's modeling process.

**David DiMarco** Neumann University
**Ryan Savitz** Neumann University

#### Math Races and Jeopardy Games

Our poster is about fun activities the authors do in class to make our math classes more enjoyable for our students and facilitate learning. Dr. David DiMarco likes to split his classes into teams and have competitive math races approximately once a week. Dr. Ryan Savitz utilizes games of mathematical Jeopardy as both fun ways to review material as well as for purposes of formative assessment. Both of these in-class activities are greatly enjoyed by our students

**Nicholas Scoville** Ursinus College
**Dominic Klyve** Central Washington University
**Jerry Lodder** New Mexico State University
**Janet Heine Barnett** Colorado State University–Pueblo
**Danny Otero** Xavier University
**Kathy Clark** Florida State University
**Diana White** University of Colorado Denver

#### Transforming Instruction in Undergraduate Mathematics Via Primary Historical Sources

Several classroom projects for use in beginning to advanced undergraduate courses in general education, geometry, trigonometry, calculus, analysis, abstract algebra, linear algebra, number theory, complex analysis and probability are being authored. Designed to capture the spark of discovery and motivate the subsequent lines of inquiry, the projects are rooted in primary historical sources, close to the initial solution of problems that would eventually find resolution in modern definitions, theorems or algorithms.

**Gabriella Harris** Towson University
**Kimberly Corum** Towson University

#### Using Investigation Activities to Incorporate Inquiry Based Learning Principles in the Classroom

Inquiry based learning (IBL) is a form of hands-on learning that incorporates active investigation, data analysis, critical thinking, and questioning. While IBL can deepen students' conceptual understanding, it can be difficult for teachers to incorporate IBL activities into the classroom. One way to make IBL more accessible is through the use of investigation activities. Using aspects of inquiry-based learning, I developed an investigation activity where students were asked to explore, discover, and interpret patterns within Pascal's Triangle and apply these patterns to binomial expansions. A pilot version of this activity was implemented in the Fall 2017 semester, which informed further revisions of the activity to better facilitate student-led investigations of numerical patterns. The revised version was then implemented the following spring semester with both middle and high school students. I will share my findings from the Pascal's Triangle lesson, including activity development, student work examples, and student reflections, and explain how investigation activities can be developed and implemented as a method of instruction.

**Adam F. Childers** Roanoke College
**David G. Taylor** Roanoke College

#### Classroom Stats: Spice Up Your Classroom with Fun, Live, Data Collection and Analysis

Studies show that students are more engaged in a statistics class when the data being used is personal, either because the data supports a cause they care about or because it is about them. In this poster, we exhibit a new, free, mobile application and website for teaching statistics that allows instructors to collect data (via fun, small, games), run experiments, and instantly analyze the results in front of the class. Instructors and students can also download the raw data to use in other statistics software such as MINITAB, R, and SPSS. The benefits of using the Classroom Stats system is that it eliminates the tedious process of manually collecting and recording data from students, and it allows the instructor to design and run meaningful experiments to teach statistical inference. We will have phones and a computer available in order to demonstrate how to use this application live for both descriptive and inferential statistics, and we will discuss our experiences of using it in the classroom.

**Heidi Hulsizer** Benedictine College
**Megan Lutz** University of Georgia
**Dione Maxwell** Loganville High School, Loganville GA
**Jonathan Templin** University of Kansas
**Laura Zielger** Iowa State

#### Creating a Learning Map for Introductory Statistics

We have developed a learning map that illustrates how students develop their ability to think about univariate, quantitative statistics. The map displays the skills typically taught in an introductory statistics course. This model hopes to illustrate the optimal learning sequences for such a course. The goal is to validate the map using standardized assessments. The map can help instructors developing course material and students in their learning process.

**Meredith Anderson** Adams State University

#### Culling Engaging Statistical Activities From the Wild: R as an Aid to Reading the News

Mathematical literacy is a crucial topic whose importance is often not given due weight, especially by those who lack the proper numeracy skills. One major instance of this deficit is in the misrepresentation and improper analysis of data sometimes appearing in the news. The populous at large is not thoroughly educated in the study of statistical methods, even though such reasoning is jointly more prominent today and potentially more accessible as well. In the math classroom, I find that student fears of mathematics can deter them from recognizing the possible biases and misrepresentations of data that can be widespread in the news. Nevertheless, this is a topic that could be interesting to a wide audience, if taught correctly and interactively. Using R, I plan to explore and uncover some of the common statistical fallacies and biases met with whilst ingesting newsprint or other common news sources. I want to illuminate these fallacies as an ancillary part of my wider math pedagogy interests and as an aid to those teaching informal and formal reasoning, especially those parts related to the interpretation of numerical and statistical data. The programming language R is well suited for this because of its numerous functions for data visualization. The fact that these sorts of things can be made dynamic and hands-on fits quite well with an interactive and discovery-based classroom, allowing the students to formulate richer questions, facilitating a deeper understanding and a better use of course time. I aim to create interactive worksheets in R so that students can see, dynamically, some of the common fallacies they are likely to encounter, as well as some of the mathematical tools which they require to intelligently and responsibly ingest news media.

**William C. Calhoun** Bloomsburg University

#### Data Science for Math Majors

Computers, digital devices and the internet have produced a huge and growing volume of data for study. The field of Data Science has arisen to derive information from data. Data scientists use and study methods to process and analyze data. Math majors have the skills needed to succeed in Data Science, but traditional math programs do not provide enough emphasis on statistics and computer programming to prepare students fully for careers in Data Science. At Bloomsburg University, we have developed a Data Science Track in the Mathematics major to address this issue.

**Michael Brilleslyper** U. S. Air Force Academy
**Kathryn Kozak** Coconino Community College
**Jenna Carpenter** Campbell University

#### Teaching Data-Centric Statistics: StatPREP at the end of Year 1

We provide an overview of StatPREP: the professional development NSF-funded program (DUE-1626337) designed to bring modern statistical thinking, professional software capability, and real data into the introductory statistics classroom. Through a series of regional workshops, with a focus on two year college faculty, StatPREP provides web-based tools and online applets to help instructors incorporate real data and modern statistical ideas into their introductory courses. We highlight the scope of the program so far, provide some preliminary results, and demonstrate some of the tools available to instructors and students. We also provide a road map for how the program will grow and the potential impact on statistics instruction.

**James Quinlan** University of New England
**Amy Deveau** University of New England

#### Outcomes and Issues from an NSF SSTEM Grant: Recruiting, Research, and Curriculum Evolution

The SUCCESS program at the University of New England was funded through the National Science Foundation Scholarships in Science, Tech- nology, Engineering, and Mathematics Program (Award #1259896). The grant has provided Maine students majoring in Science, Technology, Engineering and Mathematics (STEM) disciplines that are underrepresented or first-generation college students up to $10,000 per year for all four years at UNE. As the first SUCCESS scholar cohort advances toward graduation in Spring 2018, this poster will discuss and highlight the successes, challenges, research culture, and general outcomes of the SUCCESS scholar program at our University.

**Emma Wright** Plymouth State University

### Flipping Finite

For Finite Math and other service classes, we present a classroom structure borne out of the frustration caused by a lack of student participation. This structure blends the flipped classroom and a problem-based learning approach, and its benefits are numerous. Students attend class because they recognize the benefits of the experience. Some students are motivated to discover the material for themselves; others build curiosity about exercises they can not complete on their own. Connections between classwork and homework are more easily drawn. Students can better identify the point at which they become confused. At this poster, we discuss this classroom structure, its implementation, and its benefits to the student and instructor. We invite discussion about improving the service-math experience for both the student and the instructor.

**Mel Henriksen** Wentworth Institute of Technology

### Polishing a Flip

After a disappointing first encounter with flipping an introductory differential equations course, I employed a number of techniques to: 1) better assess student video viewing compliance, 2) encourage more class interaction in group problem solving, 3) better engage with students during in-class group work, 4) provide an array of learning tools for students of different learning styles and preferences and 5) share the resources I had developed with colleagues. Slowly drawing students into a flipped pedagogy, giving them pre-class WeBWorK assignments to encourage their timely video viewing and self-assess their understanding, and encouraging and cajoling them to engage in group work while standing at the whiteboard or window, resulted, over a two year period, in a qualitative increase in classroom engagement, classroom "energy" and ability for me to engage students in small groups, while seeing a quantitative increase in student final exam performance.

**Jill Shahverdian** Quinnipiac University

### Implementing Inquiry Using POGIL

Active learning, including activities and inquiry-based modules, has been part of my classroom for several years, with varying results. This year I have incorporated Process-Oriented Guided Inquiry Learning (POGIL) which offers a particular structure for IBL activities, and aims to address both content knowledge and process skills. This presentation will include an introduction to POGIL, my experience with POGIL, and sample activities for upper-level mathematics courses.

**Nermin Bayazit** Fitchburg State University

### Teaching Mathematics to Future Teachers: A new Layer to the Content

In this poster presentation, I will share a new addition to the curriculum to a content course that is designed for future early childhood, elementary and special education students. This is an inquiry based mathematics modeling course that is one of the three specifically designed mathematics courses for this population. In addition to the math content, I started assigning "practitioner readings" related to the content that we are covering in the class. I will share my experiences in the process and offer further recommendations.

**Danny T. Lau** University of North Georgia

### Use of a Popular Logo, the Five-cornered Star (aka Regular Pentagram) in Teaching Different Levels of Mathematics

Five-cornered star or regular pentagram is a popular design/logo that is used by many companies in presenting their ideas. We will explore the beauty of this shape in different level of mathematics. For example, in an informal geometry class for pre-service K-8 teachers, we can construct, cut and eventually fold it into a pentagonal prism to do various measurements and approximations. In a pre-calculus class, we can use it to look at various trigonometric identities and relations. In an advanced number theory class, we can use it to discuss the Fibonacci number (golden section) ratio to demonstrate why this logo is so popular in the real-world. There will be copies of regular pentagram available for participants to do some hands-on work during the poster session. Participants are most welcomed to suggest additional use of such shape in the teaching of other mathematics concepts.

**Rachel Schmitz** Towson University
**Kristin Frank** Towson University

### Students' Mathematical Modeling of One-Point Perspective Paintings

Mathematical modeling provides an opportunity for students to connect their classroom experiences to everyday life. Learners can construct mathematical models of all types of phenomena including music, fine arts, sports, movement, etc. While many modeling activities involving the fine arts focus on noticing patterns and geometric relationships, in this project we studied how pre-service and in-service secondary math teachers used systems of linear equations to understand one-point perspective. We use the modeling process described in the Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME) as a framework to understand students' modeling decisions while using systems of linear equations to model the vanishing lines and vanishing point in one-point perspective paintings. In this poster we provide student work to document how students *make assumptions* and how they *do the math* relative to those assumptions. We conclude by demonstrating that through this modeling activity, students both developed a deeper understanding of the mathematical idea of systems of equations as well as a better understanding of one-point perspective.

**Kayla Blyman** United States Military Academy - West Point
**Lisa Bromberg** United States Military Academy - West Point
**Kristin Arney** United States Military Academy - West Point

**How (NOT) to Make Discovery-Learning Assessments**

If we strive to make assessment time a time for learning, we must put great effort into creating the assessments that will challenge our students. Our discovery-learning assessments consist of four parts: a night before read-ahead focused on a new application, an in-class individual portion where students respond to short answer questions, an in-class group portion where groups of 3-4 students provide team responses to similar questions after discussion, learning, and consensus, and an individual follow-up question. We believe that this will result in better attainment of higher order learning goals, better preparation for professional collaboration, increased technology skills, and more creative excellence. To achieve these goals, we, as assessment writers, need to think critically about how to craft these non-traditional assessments. We will provide an example assessment and discuss the challenges associated with its creation. Based on these challenges, lessons learned and future considerations for assessment creation are discussed.

**Alexis Wilding** Weber State University

**Investigating Properties of Magic Matrices**

A magic matrix is a matrix whose elements are number of a magic square, that is, every row, column, and diagonal has the same sum. In this poster, we present properties of magic matrices with a new approach. Our focus is on matrices of order three but examples of high order matrices are also provided.

# General Contributed Poster Session III

## Friday, August 3, 1:00–2:30 PM, Plaza Exhibit Hall

**Tim McEldowney** University of California, Riverside

**A Better Path to Math Careers**

To help advance women in mathematics, we first need to ensure there actually are women starting careers in mathematics. However, many women graduate with a degree in mathematics without even knowing about the possible career paths in math, or being underprepared to pursue them. Last year, as a graduate student at University of California, Riverside, I decided to do something about this. Now with the support of Dr.s Po-Ning Chen and Yat-Sun Poon, graduate students and staff, we have created the Advanced Mathematics Program (AMP). AMP is a free Summer Program that prepares students for abstract algebra and real analysis, two topics which often prove to be barriers to reaching careers in math. In addition, we help students learn about what future careers they can consider, with talks from mathematicians in pure math, applied math, and math education. I will talk about how I initially designed the program and the lessons I learned in its implementation. The timing of this talk will align with completion of the second full summer of the program, providing additional opportunities to reflect on its progress and effect.

**Victoria Kofman** Stella Academy

**Girls $<3$ "Hands On Mathematics"**

Historically, girls are underrepresented in STEM. In Stella Academy, an after-school learning center, we can see, why. Boys come to us for remediation and enrichment, but, girls..., they come mostly for remediation. As soon as they develop fluency with school math, they leave saying school math is enough. As a result, in our junior high and high school enrichment classes, we have 5:1 boys-to-girls ratio. However, the situation started improving, after we opened "exploratory geometry" class, which was designed to help students of different abilities develop deeper understanding of geometry. We found, this class to be highly attractive for girls. Moreover, we were excited to learn that several girls, who never thought about STEM, after attending the exploratory geometry class, expressed interest in engineering. To meet students' interests, we created a continuation to the exploratory geometry class and began providing drafting lessons, where students learn how to create orthographic projections with the final goal of designing and printing 3D models. Our girls enjoy this class and continue attending their core-math classes even after becoming fluent with school math. So, the percentage of the girls in our enrichment classes started gradually increasing. I believe our small-scale experience with "catching" girls' attention with our "hands on" exploratory geometry lessons and gradually helping them develop interest in STEM, can be applied in other setting too: in high schools and colleges.

**Camille McKayle** University of the Virgin Islands
**Robert Stolz** University of the Virgin Islands
**Nadia Monrose** University of the Virgin Islands

### Partnerships Within and Without: Expanding the Reach of the Mathematical Sciences at the University of the Virgin Islands

The University of the Virgin Islands (UVI) is a small (approximately 2300) Historically Black College/University (HBCU) in the US Virgin Islands. Through partnerships within the university, with other universities, with the local government, and with private foundations, the Department of Mathematical Sciences is able to offer students a variety of choices within the field of mathematical sciences. A complete revamping of the mathematics major in 1999 led to the ability to offer a strong mathematics curriculum while giving students career guidance and choices within the major. Partnerships within the College of Science and Mathematics resulted in the offering of concentrations in computational science or computational biology, which resulted in an increased number of graduates studying some form of computational science in graduate school. Through partnerships with other institutions, students are able to earn a dual degree in Applied Mathematics (UVI) and Engineering (Partner Institution). This degree single handedly increased by five to six fold the number of students in the mathematics programs. We will discuss these and other partnerships through which this small department is able to offer an innovative bridge program (from high school to college), a master's program for secondary teachers, and strong and varied undergraduate programs. We will also share results from surveys of graduates, and how the feedback is utilized in guiding further revisions to the program.

**Hong P. Liu** Embry-Riddle Aeronautical University

### Use Interinstitutional Collaboration and Cyberlearning to Offer Computational Science Courses for a Computational Math Degree Program in a Small University

Embry-Riddle Aeronautical University (ERAU) is a private university with two small resident campuses, one in Daytona Beach Florida, and the other in Prescott Arizona. The mathematics department at Dayton Beach campus had been a service department for decades until the first bachelor degree in Computational Math (BSCM) was launched in 2008. In the past decade, the number of BSCM major has been relatively steady at 20-30. Moreover, about 30 students have declared BSCM as their second major. The degree program was challenged by cost-justifiable enrollments and faculty resource to offer the full set of courses in the curriculum. With the support of NSF, three computational science courses were jointly offered and shared by Adams State University, Daytona Campus and Prescott Campus of ERAU from 2014-2017. This project has been renewed and extended to six courses and four small universities from 2017-2020. The presentation will focus on the outcome of this interinstitutional collaboration and the educational technology to facilitate the blended learning for the students in multiple classrooms. If time allows, the present will share other Innovative approaches to sustain and expand the BSCM program.

**Charlie Smith** Park University

### Beautiful Integer Patterns, Version 4.0: Variations on a Binomial Theme

This talk will feature formulas for finite sums involving binomial coefficients, combining them with alternators, monomials and exponential functions. These formulas constitute one of the many fascinating facets of number theory.

**Barry C. Husowitz** Wentworth Institute of Technology

### A Machine Learning Approach to Designing Guidelines for Acute AquaticToxicity

A Machine Learning Approach to Designing Guidelines for Acute Aquatic Toxicity Dr. Barry C. Husowitz A support vector classification wrapper feature elimination approach was used to find the most relevant pairs of molecular features that adequately and accurately can predict acute aquatic toxicity. These pairs were then used to derive chemical thresholds or boundaries between chemical properties for toxic and nontoxic organic chemicals that can be used as a "rule of thumb" to design less toxic chemicals. The most relevant pairs were determined to be: Lowest Unoccupied Molecular Orbital (LUMO) and Aqueous Solubility (QPlogS), Difference between the LUMO and HOMO (dE) and Octonal- Water Partition Coefficient (QPlogo.w), and Difference between the LUMO and HOMO (dE) and Van der Waals surface area of polar nitrogen and oxygen atoms (PSA). Projected hyper planes were constructed for each pair and the following thresholds were found: for Lowest Unoccupied Molecular Orbital (LUMO) and Aqueous Solubility (QPlogS) they roughly correspond to QPlogS ¿ -1 and LUMO ¿ 1, and for Octonal-Water Partition Coefficient (QPlogo.w) vs. difference between the LUMO and HOMO (dE) they roughly correspond to QPlogo.w ¡ 1 and dE ¿ 9. This study shows how a statistical approach such as support vector machines can be applied to the rational design of chemicals with reduced toxicity.

**Zachary Beamer** University of Virginia

### Discussing Mathematical Microaggressions with Pre-Service K-8 Teachers

Dr. Francis Su introduces "mathematical microaggressions" or "microslights" as the language, behavior, and assumptions that mathematics educators use that unintentionally communicate that some students do not belong in mathematics. His article in the October/November 2015 issue of MAA Focus offers several examples of phrases commonly employed by mathematicians in the classroom that may had adverse effects on students' self-efficacy and feelings of belonging in the mathematics classroom. As an assignment for an algebra and number theory course for elementary teachers, I had pre-service teachers read and respond to this article. Students summarized the point of the article and discussed

the extent to which its suggestions are relevant to teaching mathematics at an elementary level. I asked them to suggest their own phrase or comment that should be avoided when teaching elementary mathematics. In this talk, I will present how students responded to this assignment and what suggestions they had for discussing mathematical microaggressions in a K-8 context. This presentation will include excerpts from student responses and recommendations for practitioners. Though the focus will be on discussing mathematical microaggressions with pre-service elementary teachers, this topic has broad applicability, and mathematics educators at all levels are welcome to attend.

**Derek J. Sturgill** University of Wisconsin: Stout

### Letting Teachers Notice and Wonder

Teachers hold a unique perspective regarding their own practice and the outcomes of their practice: student learning. They, in a sense, are the closest to the day-to-day actions of teaching and learning. What better way to investigate these areas than with teachers who engage in inquiry about their own practice. In this paper, I described a journey of inquiry in which four elementary school teachers engaged in research about their own practice. The story begins with a process of deciding what to study (what teachers noticed and wondered), continues to how they acted on their noticings and wonderings, and ends with what they learned from these noticings and wonderings. This journey was not a party of one. These teachers were supported by two mathematics teacher educators and were enrolled in a sustained professional development program focused on inquiry. From their journey, the four teachers grew as practitioners as well as developed as teacher-researchers. Thus, teacher professional development can begin (and can be quite successful) when teachers notice and wonder about their own practice.

**John F. Ekpe** Accra Technical University

### Relationship between Students Van Hiele Levels and the Geometric Content

Mathematical geometry thinking and reasoning in the 21st century is very important. The purpose of this study was to determine the relationship between students' van Hiele Level (VHL) and the proportions of VHL geometric content in the junior high school (JHS) and senior high school (SHS) mathematics curriculum. One hundred and seventy-four JHS 3 students, 133 SHS 1 students and 246 SHS 3 students of ages range from 14 to 20 years responded to the van Hiele Geometry Test (VHGT). The VHGT was designed to determine the classified and modified VHLs of the students. Results of this study were: (1) the geometry content of the high school mathematics for both JHS 3 and SHS 1 covered three-fifths of the items in the VH subtest 3, and for the SHS 3 the same proportion of items was covered in the VH subtest 4; and (2) only a quarter of SHS 1 students and nearly half of JHS 3 and SHS 3 students were at VH level 1, even though the geometry content covered all the items in the VH subtest 1 Suggestions for further research were made.

**J. Mealy** Austin College
**Tyler Shaw** Austin College

### Thinking Outside the Plane: teaching *not*-Euclidean geometries at pre-College levels

Our research focuses on introducing a variety of 'not-Euclidean' geometries at various pre-college levels. The 'not-Euclidean' geometries we discuss operate under axiom sets distinct from those for Euclidean geometry, but that nevertheless are straightforward to describe; further, we claim, they are appropriate for these age groups. These include various 'generalized traffic flow' schemes, 'box-house' schemes, and geometric systems with 'star-gates'. Some data, both quantitative and qualitative, from test runs of lessons involving these systems will be presented. The unifying premise of the project is that engaging in geometric ideas distinct from those of Euclidean geometry provides for a much-needed compare/contrast experience at this mathematical level of learning. It seeks to broaden students' understanding of the more general notion of geometry, as well as actually enhancing their understanding of Euclidean geometry. In so doing these lessons provide an opportunity for students to enhance critical thinking and to broaden their mathematical reasoning skills.

**Suzanne Caulfield** Cardinal Stritch University

### Reducing Student Testing Anxiety by Implementing a Three-Stage Group Testing Method

Two-stage group testing methods in mathematics have been shown to reduce student stress during exams. In Ives [1], the argument was posed in favor of two-stage collaborative group exams for just this reason. Sainsbury [2] concurs and discusses the opportunity students have to learn during the testing process, rather than being penalized for a lack of knowledge. Sainsbury also noted that regression can occur, due to group dynamics. Finally, the CWSEI [3] gives strategies on how to implement a two-stage test. However, no evidence or research has been found on the benefits of three-stage testing. This paper will highlight the rationale of three-stage testing and give preliminary examples of its success in reducing student stress during exams. One of the ways for an instructor to reliably assess student learning is to quiz the student. However, this quizzing of the student creates such levels of stress that the student cannot adequately represent themselves in the quiz. How can I find out what a student knows, if I ask them in such a stressful situation that they cannot tell me? A group testing method reduces the level of stress on the student. It also acts as an active learning opportunity for the student to recognize the gaps in their knowledge and learn from their peers. Group testing also allows the student to be assessed in a manner closer to what happens during regular class-time learning. Group exams allow the students to have time to work collaboratively and to discuss topics, in order to solidify their learning. PROBLEM STATEMENT: A three-stage testing method is implemented into a number of mathematics courses in fall 2017, to see if the level of anxiety around testing can be reduced and to improve student learning outcomes..

**Bradley J. Paynter** University of Central Oklahoma
**Elizabeth Lane-Harvard** University of Central Oklahoma

#### The Initial Development of ICAP4Calc: An Inventory of Algebra Concepts

Approximately 300,000 students enroll in mainstream Calculus I at postsecondary institutions each year. However, a significant number of students do not complete the course. In a report orchestrated by the MAA, they concluded that there is either something wrong with admittance requirements or with instruction. This project looks at the former. The purpose of this project is to develop an open-access inventory of the concepts necessary for students to succeed in a university Calculus I course. Research is currently being performed at the University of Central Oklahoma, with commitments from other universities to pilot the Inventory at a later date. The project utilizes an exploratory, mixed methods, instrument design study approach incorporating both quantitative data (inventory responses, final grades, etc.) and qualitative data (interviews). We'll present the current findings of this multi-year project.

**Caira B. Bongers** Bryn Athyn College

#### Using History to Integrate a Faith-based Mission into the Mathematics Classroom

Some faith-based institutions expect faculty in various departments to integrate the mission into the classroom. In mathematics, this can be difficult. Inspecting various mathematician's theological inquiries can build a bridge between the mathematics and theology. I provide at least three examples of parallels between famous mathematicians' approaches to mathematics and their approaches to theology. I use this springboard to ask theological questions, which can be tailored or integrated into questions posed by any institution with a theological mission. One example is that of Leibniz. His work in the development of calculus and his work /Theodicy/ show a similar perspective. He approached calculus, in part, as a solution to the problem of optimization. Optimal solutions are often finite. He described God as the great optimizer in his rationale for the existence of evil: that God made a world with as much possible goodness as could be obtained. This discussion springboards into conversations with students around why bad things happen to good people, or about the origin of evil in general. A second example is that of Pascal. In lectures on probability and expected value, the professor can connect to the well-known Pascal's Wager, and then further connect to ideas of the nature of faith and how faith is obtained or developed. I have used these discussions in my classroom and can speak to the receptiveness of students to discuss these ideas. (There are no assessments associated with these discussions, and thus there is no data to support a hypothesis of these discussions furthering their understanding of the original mathematics, or of the historical connections.)

**Alexander G. Atwood** Suffolk County Community College

#### What Should We Teach in Mathematics as Artificial Intelligence Becomes Increasingly Powerful?

Artificial Intelligence has become increasingly powerful in the past five years. New techniques such as Deep Learning Networks have been successfully implemented to make meaningful progress in difficult problems in medical diagnoses, in game playing (such as the games of Go and Poker), and in the emerging area of autonomous vehicles. Artificial Intelligence also has the real potential of transforming the workplace by powerfully augmenting human performance. In 2013, Carl Benedikt Frey and Michael Osborne, of the University of Oxford, examined the probability of computerization for 702 occupations and found that 47% of workers in America had jobs at high risk of potential automation in the next 10 to 20 years. What should we be teaching in our math courses if Artificial Intelligence will radically change the nature of employment? What skills will our students need to navigate a world in which many jobs may be transformed or even disappear because of Artificial Intelligence? How will increasingly powerful Artificial Intelligence systems change the way in which mathematics is taught in colleges?

**James R. Valles** Jr., Prairie View A&M University

#### Views on an Open Technology Policy in Mathematics Classrooms

In this poster presentation, we will discuss faculty views regarding the implementation of an Open Technology policy in mathematics classrooms. Specifically, the Open Technology policy was defined as allowing students to use any and all available technology they wished during class lecture, learning activities, and assessment. Also discussed will be the perceived benefits and drawbacks of having such a policy on student learning, understanding, and assessment.

**Kristi Karber** University of Central Oklahoma

#### Providing Mathematics Students a Transformative Learning Experience Beyond the Classroom

The mission of a metropolitan university in Oklahoma is to "help students learn by providing transformative education experiences..." As educators, we need to support this mission while fulfilling our teaching, research & scholarly activity, and service responsibilities. Oftentimes, this is easier said than done as there are only twenty-four hours in a day! One way to achieve all of the above is to integrate the students' transformative learning experience into your work beyond the classroom. We will present specific examples of how this has been accomplished and the outcomes of these projects.

**Mindy B. Capaldi** Valparaiso University

### Establishing Practices Integrating Commuter Students - Year 1

The EPIC grant focuses on the integration of STM (Science, Technology, & Mathematics) commuter students into the Valparaiso University (VU) campus community through shared undergraduate research experiences, as well as skill-building and social activities. Goals of the EPIC grant include improving recruiting and retention of STM students, increasing the number of STM graduates prepared for research careers, and increasing the number of research intensive majors within STM departments. Students are recruited with scholarships averaging $5,300 per year. This poster reports on the assessment and recruiting results of year 0, as well as the mentoring and cohort-building activities of year 1.

**Azar Khosravani** Columbia College Chicago

### Logarithmic Patterns in Classical Music

We analyzed a selection of over 500 pieces of classical music composed by Bach, Beethoven, Mozart, Schubert and Tchaikovsky, and found a surprising connection with mathematics. For each composer, we extracted the time intervals each note was played in each piece and found that the corresponding data sets are Benford distributed. Remarkably, the logarithmic distribution is not only present in the leading digits, but for all digits. We will present strategies used to comb through a significant amount of data to extract the needed information and show how we used Mathematica to analyze our data.

**Jennifer Bergner** Salisbury University

### Number Talks: A Vehicle for Understanding

As part of my latest sabbatical project I engaged several first-grade teachers and their classrooms in number talks. A number talk is a short, regularly scheduled routine that provides students with meaningful ongoing practice with mathematical thinking. The number talk leader plays the role of a scribe and puts the doing of mathematics on the class. The scribe is to write down what is shared, but is not to offer praise or critique. As such, it offers a glimpse into the way your students view mathematics. I have also used number talks during professional development experiences and in my undergraduate classrooms. I will share some of the insights I gained about reaching students and engaging them in mathematics from the sharing that happened during these experiences.

**Curtis Herink** Mercer Universiy

### Sequences with the Zeckendorf property

Zeckendorf's Theorem says that every positive integer can be expressed as a sum of distinct, nonconsecutive Fibonacci numbers in one and only one way. We therefore define a sequence to have the Zeckendorf property provided every positive integer can be expressed as a sum of distinct, nonconsecutive terms of the sequence in one and only one way. *Theorem*: The only strictly increasing sequence of nonnegative integers with the Zeckendorf property is the Fibonacci sequence 1, 2, 3, 5, 8, ... . We also give several examples of other sequences with the Zeckendorf property that violate one or more of the additional conditions of the theorem, including a sequence all of whose terms are irrational numbers.

**Grace E. Cook** Bloomfield College

### Surviving the Apocalypse with a Compass and a Straight Edge

The end is nigh! Zombies, locusts, plague! Grab your bug-out bag! Matches, check. Pocket knife, check. MREs, check. Math book, check? Can trigonometry help you fight off zombies? Will you need matrices to organize and distribute your supplies? Will parabolic motion aid you in defending the walls of the medieval castle you take over in Europe? In this research project, we examine CDC and FEMA resources along with more non-traditional sources such as Reddit, Prepper and Survivalist websites, dystopian novels, and our favorite (and not-so-favorite) zombie and post-apocalyptic movies, television shows, and video games in order to develop a mathematical plan to survive in a frightening, new world. We will make a determination of what formulas, numeric tables, and mathematical tools will be most useful depending on the resources available (electricity, batteries, solar power, etc.). We will also share various calculating and accounting methods that would work well in a variety of catastrophic situations. Future extensions of this research, which will also be discussed, include mathematical curriculum resources and a mathematical resource that could prove useful in real-world situations such as natural disasters, terrorist attacks, or war zones where resources are scarce and certain math is necessary.

**Janet Nichols** Colorado State University - Pueblo

### Colorado State University - Pueblo - 40 years and Counting

The Department of Mathematics at CSU-Pueblo has hosted MATH DAY for forty years. We serve the southeastern region of Colorado and have students participating from schools as small as 21 students in the entire high school to schools as large as 3,000 students. There have been as many as 500 students participating, but more recently around 225. This event has allowed us to network with high school teachers, as well as attract students to our campus. One of the teachers from rural Colorado has participated for 39 years. Activities for students include a team competition and an individual written exam. Teachers attend a professional development opportunity while students are taking the written exam. Other departments in The College of Science and Mathematics offer open labs and lab demonstrations. The Engineering Department also provides displays of student projects. CSU-Pueblo students help with registration, timing of bowl sessions, and proctoring and grading the written exam. High school teachers that change school districts frequently contact us so that they can bring students to our campus for MATH DAY. We feel that this has been a successful outreach activity.

**Claire Merriman** University of Illinois at Urbana-Champaign
**Emily Heath** University of Illinois at Urbana-Champaign
**Simone Sisneros-Thiry** University of Illinois at Urbana-Champaign
**Jenna Zomback** University of Illinois at Urbana-Champaign

### Summer Illinois Math Camp

The Summer Illinois Mathematics Camp is a free, week-long math day camp for middle and high school students. The program is run by graduate students, who design courses in collaboration with undergraduate teaching assistants. We will discuss how camp has expanded over four years, what we learned from changing the structure of the camp, and our plans for improving the model for future summers. The poster will include data and reactions from student pre- and post-surveys, as well as reflections from instructors and teaching assistants. Support is provided by University of Illinois at Urbana-Champaign Department of Mathematics, Illinois Geometry Lab, and our student chapter of the Association for Women in Mathematics; Mathematical Association of America Dolciani Mathematics Enrichment Grant; and the National Science Foundation.

**Axel Brandt** Northern Kentucky University
**Tanya Chartier** The Davidson Center
**Tim Chartier** Davidson College

### Taking Math to the Streets

Seeking to surprise and amuse with entertaining mathematical routines, Maths Busking has been engaging audiences on the streets of Great Britain since 2010 under the direction of Sara Santos. This year, we have worked with Davidson College undergraduates to create their own version of math street performing. Designed with both the street and K-12 classrooms in mind, many acts can be adapted for extended formats. Come learn some tricks of the busking trade!

**David R. Scott** Univ. of Puget Sound

### The 2018 SUMMA Math Teachers' Circle Workshop

The MAA will hosted a joint MAA-AIM Math Teachers' Circe at the Carriage House June 18-22. The workshop, as part of the SUMMA program at the MAA, focused specifically on teams from regions with a substantial number of students from groups underrepresented in mathematics. David Scott, one of the facilitators for the workshop will will summarize the workshop and also present a series of problems inspired by the problem of Gerrymandering. The Gerrymandering theme gives rise to arithmetic, geometric, and statistical open-ended questions that are easy to access and difficult to master.

**Soumya Bhoumik** Fort Hays State University
**Sarbari Mitra** Fort Hays State University

### $L(2,1)$-Labeling of Circulant Graphs

An $L(2,1)-$labeling of a graph $\Gamma$ is an assignment of non-negative integers to the vertices such that adjacent vertices receive labels that differ by at least 2, and those at a distance of two receive labels that differ by at least one. Let $\lambda_2^1(\Gamma)$ denote the least $\lambda$ such that $\Gamma$ admits an $L(2,1)-$labeling using labels from $\{0,1,\cdots,\lambda\}$. A Cayley graph of group $G$ is called a circulant graph of order $n$, if $G=\mathbb{Z}_n$. In this paper initially we investigate the upper bound for the span of the $L(2,1)-$labeling for Cayley graphs on cyclic groups with "large" connection sets. Then we extend our observation and find the span of $L(2,1)-$labeling for *any* circulants of order $n$.

**Colby Kelln** University of Michigan
**Sean Kelly** University of Michigan
**Justin Lee** University of Michigan

#### A Missing Entry in Sullivan's Dictionary?

Sullivan's dictionary highlights some similarities between 1-dimensional complex dynamics and 3-dimensional hyperbolic geometry. Many mathematicians attempted to use this "correspondence" to translate proofs from one field to the other. Some succeeded, some failed, but nonetheless this dictionary has driven enormous expansion in both fields since the 1980's. More recently, some mathematicians conjectured that there might be some missing entries. One of these possible missing entries is the degeneracy of Julia sets and degeneracy of limit sets of quasi-Fuchsian groups. The goal of this project is to use computer programs to generate images of degenerate Julia sets and limit sets that share similar behavior.

**Galit Eizman** Harvard University

#### Teach-Touch (Economics)

Graphs and figures are a visual way to explain correlations, functions and more theoretical and mathematical concepts. Yet, some students who still find these concpets challenging, may respond the best to three dimensional and dynamic representations. Teach-Touch (registered trademark and patent) is an innovative, touchable, three dimensional, dynamic and illustrative model, to enable a physical movement of curves and graphs in the space and clarify mathematical concepts for students. While this model can be helpful to implement mathematical concepts in all fields of study, the first focus of this model is the field of economics, as this is the first three dimensional models in this field of study, which requires a lot of mathematical understanding. Mathematical concepts and their implementation in the field of economics will be presented, using the Teach-Touch model, as: supply, demand, utility, costs and profit, income changes, exogenous and indogenous variables, movement along the curves as opposed to movement of the curves and more.

**Deborah J. Gougeon** University of Scranton

#### Student Assumptions about An Introductory Course in Business Statistics and Their Impact on Learning Outcomes

The goal here is to demonstrate empirically the impact of student attitudes on learning outcomes in *Introductory Business Statistics* courses. As a means toward this end, this study surveyed, over a period of five years, a total of 386 undergraduate students in the first class of a required *Introductory Business Statistics* course. All 386 students in the course filled out the survey resulting in a one-hundred percent response rate. Students were asked to respond to eleven questions that measured their perceptions and expectations regarding this course. For example, students were asked how much time they anticipated spending on homework assignments as well as on general study for the course, whether the course would be relevant or not to their overall business education and what their anticipated final grade would be. These factors can have a significant impact on both pedagogy and outcomes when planning and teaching an *Introductory Business Statistics* course. This study examines these attitudes and perceptions in detail in a number of ways, including making comparisons with students' previous experience with quantitative courses.

**Ge Mu** Penn State New Kensington

#### Motiving Students through Extra-Curricular Activities

To motivate students' interests in learning mathematics, I have been offering a variety of extra-curricular activities. Through Math Happy Hour, I arrange students in groups to work on projects in mathematics or math related topics, which offers students opportunities to learn or make discoveries by themselves. Via organizing the Math Club, I seek to expose students to fun topics either classical or of hot trends, e.g. blockchain and neural network, so that beyond the foundational math they are learning they can get some idea on how math is involved in the modern science and technology. Our goal is to inspire the students and encourage the enrollment in advanced math learning.

**Tracii Friedman** Colorado Mesa University

#### An Invitation to Study Mathematics: The First-year Seminar Course at Colorado Mesa University

At Colorado Mesa University, we have created a Mathematics Freshman Seminar designed to attract, engage, and retain a strong cohort of mathematics majors and minors. One of the key strengths of the course is that it exposes students to advanced mathematical thinking (beyond calculus) early in their college careers. The course also includes academic and career advising sessions and provides opportunities for interaction with upper level majors and alumni. In this poster, we will describe our motivation for introducing the Freshman Seminar course as well as the course goals; the structure and topics covered; and the pedagogy and assessments used in the course.

**Barbara O'Donovan** Saint Michael's College

### Measuring Income Inequality in a General Education or Calculus Mathematics Classroom

Income inequality is a central social justice concern, and hence excellent motivation for real-world applications in mathematics classrooms at every level. We describe the *Mathematics for Social Justice* course at Saint Michael's College, giving a specific example of one of the typical social justice projects for the course, and showing how projects can be adapted to other courses such as Calculus. The projects described focus on the Gini coefficient, a commonly used measure of income inequality. The original lesson used the trapezoid rule and Microsoft Excel to estimate the Gini coefficient for a country, while the project developed for a *Calculus I* course uses Maple to fit a power function to data and then integration to calculate the Gini coefficient. We also include readings on the Gini coefficient's role in policy formulation and advocacy.

**Solomon A. Iyekekpolor** Taraba State University
**Oyeniyi Solomon Olayinka** Taraba State University

### Use of Multimedia Technology for Effective Teaching and Learning of Plane Geometry at the Middle Basic School Level in Nigeria

In order to communicate new knowledge efficiently and appropriately in mathematics, it is necessary for teachers in Nigerian middle basic schools to be trained in contemporary teaching strategies in order to fortify the learners' experiences in plane geometry. The study discusses the use of multimedia technology for effective teaching and learning of plane geometry in middle basic curriculum since Nigerian pupils performance in mathematics has been dismal partly due to their low level of comprehending plane geometry. It highlights the introduction of television into the classroom teaching and learning processes, the benefits to teachers, students and the methodology; the cooperative efforts of teachers and the instructional media in both fortification and simplification of plane geometry education of the middle basic school pupils as well as challenges faced in the use of multimedia technology in the classroom and suggestions for overcoming the challenges and consequently improve the use of multimedia technology in the classroom are emphasized.

**Sukanya Basu** University of Toledo

### Establishing a Connection Between Julia Sets and Julia Quadratics

The French mathematician Gaston Julia (1893-1978) is famous for pioneering the study of Julia sets in the field of complex dynamics. A lesser known fact is that he also made seminal contributions to the seemingly unrelated field of algebraic geometry in the form of Julia quadratics of binary transforms. An interesting question to ask both from a historical and a mathematical point of view is: Are Julia sets related to Julia quadratics? In my poster, we explore this question and come up with an interesting affirmative answer.

**Lisa J. Mueller** University of Kentucky

### A Student's Declassified Grad School Survival Guide

Graduate school: the name itself is very intimidating and is usually only associated with the cream of the crop. Many people have heard horror stories of how hard graduate school is compared to undergrad and how many people do not make it past their first or second year. Having just finished my first year of graduate school, I have already had those thoughts of, "Is this really worth it?" "This is so hard!" "How am I ever going to get through this?" After befriending many graduate students, talking to many professors, and getting advice from advisors, I have discovered many key components to keeping a level head through the difficulties of graduate school. I am here today to share with you the knowledge I have learned over the past year from my experience at the University of Kentucky mathematics graduate department - my ups and downs, the best and worst advice I received, and overall, the reason why graduate school was the best decision I ever made.

# Graduate Student Session

## Great Talks for a General Audience

### Saturday, August 4, 1:00–5:00 PM, Governer's Square 9, 10, 14

**Kevin Bombardier** University of Iowa

#### Atomic Domains with a Small Number of Atoms

The mathematical system of the integers has many useful properties. One of these is unique factorization. For example, we can write the number 14 in a unique way: $14 = 2 * 7$. However, the numbers 2 and 7 cannot be factored into "smaller pieces" in a nontrivial way. So in this sense, they could be called atoms of this mathematical system. Other mathematical systems usually do not have all of the nice properties that the integers do. However, some useful properties can be salvaged in certain cases. An atomic domain is a special mathematical system where its members have a factorization into a product of atoms. We will discuss some important cases of these atomic domains that usually will contain only a small number of atoms.

**Elizabeth Ayisi** Ohio University

#### Visualizations of Complex Valued Functions and Mappings Using GeoGebra

In this presentation, I discuss some curriculum materials for teaching college level complex variables and validation of the visualization of complex valued functions and mappings using GeoGebra. I will give some examples of visualization for exponential, Joukowsky, power, and logarithmic functions, stereographic projection, and modules of functions.

**Fatemeh Norouzi** Morgan State University

#### The Nonlinearity of Multivariate GARCH(p,q) and Pink noise

In this paper, we aim to use the Multivariate Generalized Auto Regressive Conditional Heteroskedasticity (MGARCH) model which have been using a lot in forecasting the conditional volatility of time series. In order to study the ability to reproduce power law statistics, the probability density function (PDF) and power spectral density (PSD), we investigate several cases of the nonlinear MGARCH(p,q) such as Bivariate GARCH(1,1) and Univariate GARCH(p,q). For this investigation, we are exploring derivations of stochastic differential equations from nonlinear MGARCH(p,q) processes. By finding the similar and general class of stochastic differential equations, we will verify the nonlinear modifications to see the power law distribution and power spectral density of the pink noise.

**Ryan Moruzzi** University of California, Riverside

#### The Weyl group action on fundamental weights

In the study of Lie algebras, which are vector spaces equipped with an operation called a bracket, we try to classify their representations. For simple Lie algebras, the finite dimensional irreducible representations are classified by fundamental weights. The fundamental weights not only play an important role in the case of simple Lie algebras, but also with infinite dimensional Lie algebras. Fundamental weights can be viewed as elements of an inner product space, such as Euclidean space, and we can apply simple reflections to each element. The collection of all simple reflections generates what is called the Weyl group, and we will explore how the Weyl group interacts with the fundamental weights. This work leads to a problem I am currently investigating.

**Lauren M. Nelsen** University of Denver

#### Rainbow Spanning Trees in Graphs

Networks, or graphs, are useful for studying many things in today's world. Graphs can be used to represent connections on social media, transportation networks, or even the internet. Because of this, it's helpful to study graphs and learn what we can say about the structure of a given graph or what properties it might have. It turns out that linear algebra is a valuable tool in the study of graphs. There are matrices that we can associate with graphs, and using techniques from linear algebra, these matrices can give us valuable information about the structure of graphs. A rainbow tree in an edge-colored graph is a connected subgraph without cycles where each edge is a different color. In this talk, we'll look at some of the interesting connections between linear algebra and graph theory, including how we can use eigenvalues to get a bound on the number of rainbow spanning trees in an edge-colored graph.

**Simone Sisneros-Thiry** University of Illinois at Urbana-Champaign

### Quotients of sums of distinct powers of three

Let $S$ be the set of positive integers that have base 3 representations using only the digits 0 and 1. We can see that $7 = 2(3) + 1$ is not in $S$, but $7 = \frac{3^3+1}{3+1}$ is a quotient of elements of $S$. Which integers can be written as quotients of elements of $S$? Athreya, Reznick, and Tyson have proved a necessary condition for an integer to be a quotient of elements of $S$, but it is not quite sufficient. The smallest integer for which the condition is not sufficient is 529. We will discuss computational results and progress towards categorizing this set of integer quotients.

**Alexis Doucette** Stevens Institute of Technology
**Alicia Muth** Stevens Institute of Technology

### K-Component Order Neighbor Connectivity: A Generalization of Domination

The component order connectivity of a graph, useful in measuring the vulnerability of networks, is the number of nodes that need to be removed from the graph in order to produce a component of order less than some given threshold. The instance where the threshold value is one, known as domination, has been previously studied. The domination number, the minimum order of a set of nodes with the property that every other node of the graph is adjacent to at least one node in the set, is an 'exceptional invariant,' meaning it can increase or decrease depending on certain conditions. Our goal is to see which properties of domination carry over to higher threshold values and which do not.

**Nelson Carter** University of Houston-Clear Lake

### Tertiary Calculus Curricula: A Literature Review

From departmental collaboration in developing a common syllabus, to instructors exercising academic freedom to teach the way they want, there are different calculus curricula taught in colleges and universities throughout the United States. How are they the same and how are they different? In this paper, we will investigate them by putting these curricula into categories to compare their similarity and differences. A full review of literature about calculus curricula, their development, and research pertaining to effectiveness will be provided.

**Anae Myers** Florida Atlantic University

### If you won't moan and groan, Then I won't drone in (or about being strictly) monotone.

If your initial thoughts on monotone sequences is that they are somehow uninteresting, think again. In this talk, we explore crucial properties of monotonicity which have inspired generalizations introduced by Tikhonov with applications to the convergence of some number series.

**Bryan Carrillo** University of California, Riverside

### Axially Symmetric D-solutions to the Steady Navier-Stokes Equations

One open question in the study of the steady 3-dimensional Navier-Stokes equations is if the only solution with finite Dirichlet integral and vanishing condition at infinity is the trivial solution. Several partial results have been proven under various assumptions on the solution, such as assuming certain integrability conditions or assuming the solution decays at infinity at a certain rate. We will explore a certain class of solutions, called axially-symmetric D-solutions, and discuss some results about these solutions.

**Tim McEldowney** University of California, Riverside

### Hilbert and Jacobson Module Equivalence

Jacobson and Hilbert rings were independently discovered in the early 50s and shown to be equivalent for commutative rings. Since the turn of the century, new structures called Hilbert modules and Jacobson modules were independently created. I will show these modules are equivalent for finitely generated modules. Then talk about my work on building a Noetherian characterization for Hilbert/Jacobson modules.

**Jeremy L. Thompson** CU Boulder

### On performance and portability for generic finite element interfaces

One of the challenges with high-order finite element and spectral element methods is that a global sparse matrix is no longer a good representation of a high-order linear operator, both with respect to the FLOPs needed for evaluation and the memory transfer needed for a matrix-vector multiply. Thus, high-order methods require a new operator description that still represents a linear (or more generally non-linear) operator. libCEED is an extensible library that provides a portable algebraic interface and optimized implementations suitable for high-order operators. libCEED's operator description is easy to incorporate in a wide variety of applications, without significant refactoring of the discretization infrastructure. We introduce the libCEED API and compare performance to native implementations in production software, such as Nek5000 and MFEM.

**Otto Shaw** Ohio University

### Learning Number Theory by Inquiry

Come learn what inquiry-based learning looks like in a university mathematics course. We will share our experiences involving classroom activities, assignments, and group projects that were incorporated in a number theory course at Ohio University. We will provide glimpses of student coursework from both a student and teaching assistant's perspective. Leave this session with the mindset of engaging in inquiry-based mathematics learning at both the school and college levels.

**Deependra Budhathoki** Ohio University

### Making Mathematics Beautiful

The nature of mathematics is always a burning issue for discussions among individuals—those who are directly associated with mathematics/education and those who are not. Despite having several real-life related aspects, mathematics teaching and learning around the world is still a challenge for the majority of stakeholders. The decreasing numbers of students majoring in mathematics and decreasing achievement level of students in high school are some examples. People mostly find mathematics demotivating with its abstract and philistine aspects. Here, I discuss how mathematics is beautiful and what can bring beauty in mathematics. I start with discussions made by different authors about aesthetic components of mathematics and later continue Appreciative Inquiry (AI) as the one that provides beauty to mathematics teaching and learning. Based on my M.Ed. thesis finding, I explain how Appreciative Inquiry based practices help students to change their mindset about mathematics learning and motivate towards it.

**Sun Young Ban** Teachers College Columbia University

### The Influence of Teaching Instruction and Learning Styles on Mathematics Anxiety in College Mathematics Course

The existing literature body has suggested that particular instructional approaches may be part of the problem with respect to students' Mathematics anxiety (MA). Besides that, depending on students' learning styles, teachers may expect that students actively participate in their own learning with less math anxiety in either Lecture Classroom Model (LCM) or Inquiry-Based Learning Classroom Model (IBLCM). The purpose of this study is to investigate the influence of two different instructional models on mathematics anxiety, interacting with Kolb's (1984) four different learning styles—accommodators, divergers, assimilators, and convergers. The author intends to observe how teachers' instructional approaches and students' different learning styles are associated with students' math anxiety and academic performance. In this study, 150 students in a community college's mathematics classrooms participated both in abbreviated mathematics anxiety rating scale (A-MARS) test and Kolb's Learning Styles Inventory (LSI) before and after the 15-week mathematics classes. From the findings, accommodator-type learners showed consistent decrease in MA with IBLCM. On the other hand, the diverger-type learners consistently showed increase in MA when taught with the IBLCM. The assimilator type learners showed consistently strong decrease in MA when they were taught through LCM. Lastly, converger-type learners generally showed decrease in MA levels in IBLCM. The author concludes that students learning styles have its preferred instructional models with respect to their math anxiety.

**Harman P. Aryal** Ohio University

### Learning Number Theory by Inquiry

Come learn what inquiry-based learning looks like in a university mathematics course. I will share my experiences involving classroom activities, assignments, group discussions, group projects, and presentations that were incorporated in a number theory course at Ohio University during the spring 2018. I will provide glimpses of student coursework from a teaching assistant's perspective. Leave this session with the mindset of student engagement in inquiry-based mathematics learning at both the school and college levels.

**Katrina Johnson** University of Utah (Summer 2018) BYU-Idaho (Fall 2018)

### Modeling Fat Expansion

Chronic inflammation is associated with obesity and linked to the development of insulin resistance and other metabolic disorders. Fat cells, or adipocytes, are the primary cell type in fat tissue, or adipose tissue. Fats, or lipids, are one of the three major sources of energy we consume. Adipocytes are responsible for limiting the exposure of other tissues to lipid accumulation during the fed state and providing energy by releasing lipids during fasting. Adipose tissue expands by increasing the size of adipocytes, hypertrophic expansion, and/or the number of adipocytes, hyperplasic expansion. We developed a mathematical model to study the population dynamics of adipocytes in response to energy storage demands. We use this model to explore the hypothesis that hypertrophy is the short-term response to energy storage demands and hyperplasia is an additional response to long-term energy storage demands. The addition of immune cells, macrophages, to this model allows the study of interactions in the coordinated development of obesity and chronic inflammation.

**Katrina Morgan** University of North Carolina at Chapel Hill

### How Geometry Affects the Point-wise Decay Rate of Waves

In flat space, solutions to the wave equation decay infinitely fast. This can be seen by the Sharp Huygens' Principle, which says that if the initial data is compactly supported (i.e. vanishes outside of some closed ball) then the solution also vanishes at large enough distances for any given time. It was shown in Tataru 2013 that on a geometry that is curved but becomes flat at a rate of $r^{-1}$, waves decay at a rate of $t^{-3}$. Here $r$ indicates spatial distance from the origin. The current work examines what happens to the decay rate of waves when the geometry is curved but becomes flat at a rate of $r^{-k}$ for $k > 1$. The techniques used in this ongoing work, including resolvent and local energy estimates, will be discussed. but no familiarity with these tools will be assumed.

**Rebecca S. Terry** University of Utah

### Eco-evolutionary Dynamics of Cooperation

An organism's phenotype is defined as the expression of its genetic material. This expression may change under different environmental conditions. The ability to alter one's phenotype by turning on or off specific genes in response to changes in one's environment is known as phenotypic plasticity. Cooperation in certain microbial species is an example of phenotypic plasticity whereby some individuals express particular genetic machinery to produce a resource available to the entire population while others fail to express that same machinery but benefit from consumption of the resource without the cost of its production. Depending on the availability of the resource in its environment, an individual may switch from an expressing to a non-expressing state or vice versa. We develop mechanistic models to explore the ecological and evolutionary dynamics of cooperation in social microbes under varying environmental conditions, and examine whether there exists an evolutionarily stable switching strategy.

**Nicola Mulberry** Simon Fraser University

### Entwining Research Expertise, Calculus Knowledge, and Teaching Skills

As a grad student in mathematics, I not only perform course work and research, but am also employed as a teaching (TA) or research assistant (RA), neither of which necessarily coincides with the research I pursue for my thesis. I had the unusual opportunity to become an RA for an open educational resource (OER) development project on the teaching of calculus. In my presentation, I want to excite undergrads about continuing on in mathematics in three ways: (1) Draw attention to the synergistic relationship between math teaching and research. (2) Showcase the graphical resources that I developed to support math concepts, and the undergrad student slides that were created in conjunction with my RA supervisor Dr. Petra Menz. (3) Discuss the importance of OER for math education and its role in the Free and Open Software movement. I want to pass on the message that undergrads who aim for a career in academia need to actively pursue RA-ships that enrich their grad student experience. Most grad students are familiar with the typical drudgery of being a TA and marking hundreds of papers, but being an RA on the OER project enriched my editorial skills, technical writing skills and document management skills with LaTeX that all transfer to my thesis work. Furthermore, since a critical study of the material was required, my involvement in this project prepares me for teaching a full calculus section, which is a plum TA position for any grad student.

**Hanson Smith** University of Colorado at Boulder

### Ramification in the Division Fields of Supersingular Elliptic Curves and Sporadic Points on Modular Curves

An abelian group is an object that behaves like the integers under addition. In a general abelian group; however, we can have elements of finite order. In other words, we can add an element $P$ of our abelian group to itself $n$ times and obtain $0$. Symbolically,

$$\underbrace{P + P + \cdots + P}_{n-\text{times}} = 0.$$

A nice metaphor for the idea of finite order is a clock. If we think of 12 o'clock as 0 and we add 1 hour 12 times, we will be back at 12 o'clock or 0. An abelian group that is of interest to number theorists is the group of points of an elliptic curve. In addition to the abelian group structure of their points, elliptic curves also have a geometric structure. The interplay of the group structure with the geometric structure has been an extremely fruitful area for mathematicians. We will focus on the points of finite order on an elliptic curve. Our investigation will concern how complicated these points can be. Under suitable hypotheses, we will show that points of finite order on an elliptic curve are as far away from being rational numbers as possible.

**Julian DeVille** Eastern Kentucky University

### 2D Local Krawtchouk Descriptors for Comparison of Tissue Microarray Images

It is known that image comparison can prove cumbersome in both computational complexity and runtime, due to factors such as the rotation, scaling, and translation of the object in question. Due to the locality of Krawtchouk polynomials, relatively few descriptors are necessary to describe a given image, and this can be achieved with minimal memory usage. Using efficient computation of Krawtchouk moments to produce local descriptors, we are developing software which can query an image and compare similar patterns locally across a potentially large database. These local descriptors are rotation, scale, and translation invariant to ensure the accuracy of results, and following a query,

significant results are ranked by likelihood of the match. Our software has been tested on images from the Stanford Tissue Microarray Database.

**Brady A. Tyburski** Colorado State University

### Cheese and Tensors

Algebra invents numbers needed to solve equations (e.g. the imaginary numbers were invented to solve $x^2 = -1$). How many possible numbers can we invent? What if the numbers must commute? Be associative? Satisfy other properties? To find out, we count multiplication tables. Geometry can then be used to reduce our questions to optimization problems no harder than the ones solved in a calculus class. Along the way, we encounter objects called tensors and discover that cheese pairs nicely with a good, aged tensor.

**Anna Seitz** Iowa State University

### The Cantor Set: Patterns and Paradox

The ternary Cantor set is a fractal resulting from an iterative deletion process performed on the unit interval. Containing an uncountably infinite number of points, the resulting set is a novelty as it achieves Lebesgue measure zero. In this talk, we will build a general understanding of the Cantor set through geometric construction, base-three representation, and an iterated function system. We will then explore the paradox of the Cantor-Lebesgue function, which is monotone increasing but has a derivative of zero almost everywhere.

# PosterFest 2018

## PosterFest

### Friday, August 3, 3:00–4:30 PM, Plaza Exhibit Hall

**Holly Attenborough** University of Wisconsin-Platteville

### Exploring Math Explorations

Why does satisfying a general education math requirement have to be so daunting? An aim of a liberal arts math class is to remove the daunt, yet still provide an excellent foundational math course where students gain and improve problem solving skills. This fall semester, I administered a pre- and post-test evaluating self-efficacy, confidence, mindset, and ability to define mathematics in two sections of Calculus I and one section of Mathematical Explorations (a liberal arts math class). The purpose of this study was to investigate the question: Does the University of Wisconsin-Platteville's liberal arts math class have a positive impact on attitude, self-efficacy, and growth mindset in regards to mathematics? My poster will present an evaluation of the pre- and post-test as an effective tool for answering this research question and discuss any preliminary results.

**Kim Spayd** Gettysburg College
**Ellen Swanson** Centre College

### Nonclassical Solutions for Three-Phase Flow in Porous Media

We consider a strictly hyperbolic partial differential equation model for three-phase flow in porous media and solutions of its associated Riemann problem. Previous work has showed that classical solutions of such a system, in the absence of gravity and capillarity, include rarefaction waves and Lax shocks. By incorporating capillary pressure, as given by thermodynamically constrained averaging theory (TCAT), the model includes dynamic dissipation and dispersion terms. This matches a previously studied framework in which nonclassical solutions, involving undercompressive shocks, can be uniquely determined with an entropy dissipation function. In this poster, we present the regularized model and a dissipation function which is used to identify saturation initial conditions that generate nonclassical solutions. Numerical simulations confirm the presence of these solutions.

**Kyle Czarnecki** UW-Platteville

### Arithmetic Functions of Higher Order Prime-Indexed Primes

The Sieve of Eratosthenes (SoE) is an ancient method of extracting the set of prime numbers from the set positive integers. It is well known that the prime numbers are intimately connected to the Riemann zeta function, and that many interesting results (e.g. the Prime Number Theorem) follow from the study of the Riemann zeta function. It is lesser known that applying the SoE to the index of the prime numbers will result in the so-called set of prime-indexed primes $\{3, 5, 11, 17, 31, ...\}$. More generally, the application of the SoE $n$-times will yield the set of $n$'th order prime-indexed primes. This poster discusses the corresponding Beurling zeta functions and some recent results on a few arithmetic functions attached to these zeta functions.

**Richard G. Ligo** Gannon University

### Investigation of a Nonuniform Knot Energy

A *knot energy* is a real-valued functional whose domain is the space of knot embeddings into Euclidean space. The Möbius energy is one of the most interesting of these functionals. Using it as a starting point, we define the energy of a so-called "weighted knot"—a pair of a knot and a weight function. After investigating the fundamental properties of this new energy, we use the calculus of variations to better understand how a weight function affects its value.

**Alyssa Hoofnagle** Wittenberg University

### Hamiltonicity of Bipartite Graphs Generated from Subsets

Determining whether a graph is Hamiltonian is not a simple nor quick task, even for computers. The search for Hamiltonicity is a popular exercise, and the process can utilize many different algorithms to do so. It is even difficult to determine the Hamiltonian cycle within some Hamiltonian graphs. We will consider a particular bipartite graph, $G_{n,m,k}$, whose vertices consist of $m$-subsets and $k$-subsets of an $n$-element set (where $k < m < n$ represent the cardinality of the subsets) and edges are defined by set inclusion. For select choices of $n$, $m$ and $k$, we will show that $G_{n,m,k}$ is Hamiltonian and give an algorithm for finding a Hamiltonian cycle. This work is in the preliminary stages and is accessible to undergraduate students.

**Hasala Senpathy K. Gallolu Kankanamalage** Roger Williams University

### Point-wise Dissipation of Systems with Delays

Systems with delays are highly sophisticated in nature since they require infinite dimensional spaces to describe evolution. However, in Lyapunov based stability, one can introduce point-wise dissipation constants to describe decay of trajectories. It is important to study the theoretical nature of this duality. Proper explanation can unify existing Lyapunov theories developed for control systems with delays. We analyze the nature of this problem for classical delay systems with possible extension to control systems.

**Nathan C. Morris** Carthage College
**Haley A. Yaple** Carthage College

### Political Polarization: Voting Patterns in the Senate

It seems from the current political climate that partisanship is on the rise, but how can this be quantified? In this work, we investigate partisanship in the the U.S. Senate using methods from a recent paper which studied the U.S. House of Representatives [Andris et al. 2015]. By comparing agreement both within and between parties, we create a network structure defined by vote records. An analysis of this network highlights the range of Senator behaviors and allows us to track changes in partisanship, both on the individual and party level, over decades of vote history.

**William Cipolli** Colgate University

### Prediction and Feature Selection via Smoothed Polya Trees

This work extends the supervised learning classification model of Cipolli and Hanson (2018) to prediction by smoothing Polya trees via computationally efficient Householder transformations. The proposed approach provides a very simple and efficient way to implement Bayesian nonparametric prediction as there are no kernel tricks or initialization steps in training the model.

**Sedar Ngoma** SUNY Geneseo

### On a Space-dependent Inverse Source Problem for a Parabolic Equation

We consider an inverse source problem for a parabolic partial differential equation subject to an integral constraint and a final time overdetermination in a domain $\Omega$ of $\mathbb{R}^d$, $d \geq 1$. The source function depends on the space variables only. We show the existence, uniqueness, and stability of classical solutions in Hölder spaces. We use the finite element discretization to implement an algorithm that can be used to approximate the solution of the inverse problem. Our numerical results show that the proposed scheme is a reliable procedure for approximating the solution of the inverse source problem.

**Tyler Skorczewski** University of Wisconsin Stout

### A Dynamic Energy Budget Model of Ornate Box Turtle Shell Growth

Many aspects of box turtle development depend on size rather than age. Notable examples include sexual maturity and the development the fully closing hinge in the shell that allows box turtles to completely hide in their shells. Thus, it is important to understand how turtles grow in order to have a complete understanding of turtle biology. Previous studies show that turtle shell growth behaves in a logistic manner. These studies use functional models that fit the data well but do little to explain mechanisms. In this work we use the ideas found in dynamic energy budget theory to build a model of box turtle shell growth. We show this model fits the data as well as previous models for Ornate Box Turtles *Terrapene Ornata Ornata* but also offers explanations for observed phenomena, such as maximum sizes and the appearance of biphasic growth, based on mathematics that is not seen in other models.

**Katrina Morgan** University of North Carolina at Chapel Hill

### The Effect of Metric Behavior at Spatial Infinity on the Pointwise Decay of Waves in the Asymptotically Flat Setting

The current work examines the effect of the rate at which a geometry becomes flat on the pointwise decay rate of waves. This work is motivated by the proof of Price's Law in Tataru 2014. Price's Law is a conjecture from General Relativity (GR) predicting a $t^{-3}$ pointwise decay rate for solutions to the wave equation on the Schwarzschild spacetime (i.e. the geometry of space due to a stationary black hole predicted by FR). Tataru established this decay rate for a class of spacetimes that includes the Schwarzschild metric. The behavior of the spacetime geometry at spatial infinity appears to play a decisive role in establishing the expected decay rate. The primary tools that will be explored in the poster are resolvent and local energy estimates. The poster will focus on the role that the behavior at spatial infinity plays in the proof and discuss how changing the rate at which the geometry tends to the Minkowski metric is expected to affect the argument.

**Emily Hendryx** University of Central Oklahoma
**Beatrice Riviere** Rice University
**Craig Rusin** Baylor College of Medicine

### Subset Selection with an Extension of DEIM

Index selection via the discrete empirical interpolation method (DEIM) can be used to identify representative subsets of data from a larger data matrix. However, the rank of the data matrix limits the number of representatives that can be in the identified subset–an issue, for instance, when the number of classes present in the data is greater than the number of features observed. This work presents a novel extension of DEIM to allow for the selection of additional representatives along with experimental results that support the use of such an extension.

**Sun Young Ban** Teachers College Columbia University

### The Effect of Classroom Discourse in Inquiry-Based Learning on Teaching and Learning Mathematics

Current mathematics curriculum calls for the effective teaching that students listen and explain their mathematical ideas in inquiry-based learning (IBL). In this proposal, mathematics discourse involves students' engagements where students discover and construct knowledge through their own experiences and inquire mathematics problems, and as a result, they have a variety of ways of solving problems. In this study, a community college's mathematics classrooms were observed where the instructors implemented questioning techniques to improve students' mathematical discourse and their justification. In this article, the author provides a comprehensive review of how a mathematics teacher prepares to ask different kinds of questions that help students to develop their reasoning behind processes of problem-solving. The author assesses the different kinds of questions around the inquiry-based learning themes that promote mathematical discourse in a classroom and that enable students to justify their mathematical ideas. The author focuses on how students make connections in their mathematical thinking based on the teachers' questioning strategy, think-pair-share strategy, and classroom discourse. From the findings, the author concludes that the effective questioning in a IBL classroom inspired students' analysis, synthesis, interpretation, and critical thinking on their mathematical justification leading to a problem solution.

**Xiang Ji** Penn State New Kensington

### Constructing Q-algebroids from Q-groupoids

It is well-known that the infinitesimal object of a Lie groupoid $G \rightrightarrows M$ is an associated Lie algebroid $(A, [\cdot, \cdot], \rho)$ over $M$, which generalizes the relation between a Lie group and its associated Lie algebra. In the category of graded manifolds, the groupoid and algebroid objects equipped with certain homological vector fields, namely the $Q$-groupoids and $Q$-algebroids, are also defined. In this presentation, we show that a similar result holds for $Q$-groupoids and $Q$-algebroids, i.e. the infinitesimal object of a $Q$-groupoid $(\mathcal{G}, \phi)$ is a $Q$-algebroid $(\mathcal{A}, \Phi)$.

**Jennifer Zakotnik-Gutierrez** University of Northern Colorado

### Developmental Mathematics Reform: Analyzing Change at an Urban Community College

Each year, thousands of students entering community college find that they are underprepared for college-level mathematics and must complete some number of pre-college level courses before they can fulfill their college mathematics requirement (Bailey, Smith Jaggars, & Jenkins, 2015; Scott-Clayton, 2012). Unfortunately, many of these students never make it into, let alone through, the college-level mathematics courses required for their academic major. In fact, roughly half of the nearly 44% of students referred to developmental mathematics do not successfully make it into the first relevant college-level course (Bailey, Jeong, & Cho, 2010; Snyder, de Brey, & Dillow, 2018). Given that the student population at community colleges is predominantly underrepresented groups who have high risk characteristics (Mesa, Wladis, & Watkins, 2014; Snyder et al., 2018) and these same populations are referred disproportionately to developmental education (Bailey et al., 2015; Engstrom & Tinto, 2008; Mesa et al., 2014), the current state of developmental mathematics is an inequitable disservice to students. The purpose of this proposed study is to provide an account of one community college's developmental mathematics redesign as they implement a corequisite model college algebra course. Using Gutierrez's (2009) equity framework and Tinto's (1975) model of persistence I plan to examine the experiences of students, instructors, and administrators, employing activity theory to analyze the interactions and contradictions within and between the three groups.

**Xue Gong** University of Wisconsin-Stout

### A Preliminary Study of Practices to Reinforce Algebra Skills for Calculus

Mastering algebra skills is crucial for students to succeed in a Calculus I course. There are students who generally understand those calculus concepts presented to them but lack the algebra skills to reach their full potential. This study focuses on the effects of increasing the level of preparation of students in a Calculus I course by identifying those students with weak algebra skills and providing assessment and intervention: repetitively testing on specific algebra skills until students have fully mastered these skills. Individual student's performance was monitored throughout the course of the semester, and a questionnaire was given to students at the end of the semester on their perceptions of the benefits of the assessment and intervention. Students from two sections of Calculus I ($N = 45$), taught by the same instructor, participated in this study. Preliminary results will be reflected upon so to devise practices to help students succeed in calculus.

**Nicole Durant** University of Colorado Denver

### Expectation Maximization Algorithm to Identify Genetic Subgroups in Extremely Large Samples

Expectation-maximization (EM) algorithms are widely used for estimating parameters of mixture distributions. In precision medicine, identifying mixture distributions can help identify subgroups of patients with different disease susceptibilities or treatment responses. Sometimes these subgroups depend on underlying genetic information. EM-algorithms require the estimation of additional parameters, which can decrease power to detect underlying subgroups. In our algorithm, we reduce the number of parameters and increase power to detect subgroups by conditioning on genetic information and making several simplifying assumptions. First, we assume that the subgroup only occurs on the nonreference genetic background. Second, we assume that the mean of one of the nonreference genetic subgroups equals the mean from the reference genetic background. Across a variety of simulations, these simplifications increase the power to detect genetic subgroups and time to convergence of the algorithm. Given millions of genetic variants, our method enables the efficient identification of genetic variants with subgroup effects for prioritization in further research advancing precision medicine.

# Index of Speakers