Selecting the Correct Operation

For each of the problems below, determine whether addition, subtraction, multiplication, or division is required to answer the question.

1. Mary and Jose are building a brick walkway. They have 35 bricks, 7 of which are unusable. How many usable bricks do they have?

2. Mary and Jose are building a brick walkway. They want to have a walkway that has 7 bricks in each row and 35 rows. How many bricks do they need?

3. Mary and Jose are building a brick walkway. They have 35 bricks and want a walkway that is 7 bricks wide. How many rows of bricks can they make?

4. Mary and Jose are building a brick walkway. They have 7 bricks and purchase 35 more bricks. How many bricks do they now have?

Creating Problems with a Certain Operation

1. Make up a word problem requiring addition to answer the question.

2. Make up a word problem requiring subtraction to answer the question.

3. Make up a word problem requiring subtraction for which the answer is a negative number.

4. Make up a word problem requiring multiplication to answer the question.

5. Make up a word problem requiring multiplication for which the answer has units of square feet.

6. Make up a word problem requiring multiplication for which the answer has units of dollars.

7. Make up a word problem requiring division to answer the question.

8. Make up a word problem requiring division to answer the question and using the numbers 7 and 105.

9. Make up a word problem requiring division to answer the question and using the numbers 53 and 223.

10. Make up a word problem requiring division for which the answer is a positive number less than one.

11. Make up a word problem requiring division for which the answer has units of miles per hour.

Develop Comprehension

Number Sense

1. The sum of two positive numbers is 10. Which of the following answers are not possible for one of the numbers? There may be more than one answer.

    a. -3   b. 1/2   c. 11   d. 3.7   e. 7

2. The difference between two positive numbers is 10. Which of the following answers are not possible for one of the numbers? There may be more than one answer.

    a. -3   b. 1/2   c. 11   d. 3.7   e. 7

Percent Mixture Problems

3. A solution that is 10% salt is mixed with a solution that is 20% salt. Which of the following are not possible for the concentration of salt in the resulting mixture?

    a. 10%   b. 15%   c. 18%   d. 20%   e. 25%

4. Pure water is added to a solution of sugar and water that is 20% sugar. Which of the following are not possible for the concentration of sugar in the resulting mixture?

    a. 10%   b. 15%   c. 20%   d. 25%   e. 30%

5. A 100-gram solution is 15% sugar. How many grams of sugar are in the solution?

6. A 100-pound bag of wheat and oat grain is 40% wheat. How many pounds of oat grain are in the bag?

7. A 200-liter solution of fruit juice and water contains 10% fruit juice. How many liters of water are in the solution?

Value Mixture Problems

8. A grocer mixes peanuts that cost $4 per pound with almonds that cost $8 per pound. Which of the following answers are not possible for the cost of the mixture? There may be more than one answer.

    a. $4.50   b. $3.75   c. $9.50   d. $6.25   e. $6.00

9. A 10-pound bag of coffee contains coffee beans valued at $7 per pound. What is the total value of the 10-pounds of coffee beans?

10. A 5-pound box of candy has a total value of $30. What is the cost per pound of the candy?

Uniform Motion Problems

11. Lois and Michael begin at the same time and walk toward each other on a straight road that is 2 miles long. Lois walks at a greater rate than Michael.

    a. Is the distance walked by Lois less than, equal to, or greater than the distance walked by Michael?

    b. Is the time walked by Lois less than, equal to, or greater than the time walked by Michael?

    c. What is the total distance traveled by Lois and Michael?

12. Morgan and Emma ride their bikes from Emma's house to the store using the same route. Emma bikes faster than Morgan. Morgan begins biking 5 minutes before Emma begins.

    a. When they reach the store, is the distance biked by Emma less than, equal to, or greater than the distance biked by Morgan?

    b. When they reach the store, is the time spent biking by Emma less than, equal to, or greater than the time spent biking by Morgan?

13. Keith and Jennifer started at the same time and rode toward each other on a straight road. When they met, Keith had traveled 15 miles and Jennifer had traveled 10 miles. Who had the greater average speed?

14. Two children are returning from school to home. Carla runs half the time and walks half the time. James runs half the distance and walks half the distance. Assuming the children run at the same rate and walk at the same rate, which child arrives home first?

15. Suppose you have a powerboat with the throttle set to move the boat at 8 mph in calm water and that the rate of the current of a river is 4 mph.

    a. What is the speed of the boat with the current?

    b. What is the speed of the boat against the current?

16. The speed of a plane is 500 mph. There is a headwind of 50 mph. What is the speed of the plane relative to an observer on the ground?

17. The rate of a current in a river is x mph, and the rate of a boat in still water is y mph. How can you represent the rate of the boat going down the river? How can you represent the rate of the boat going up the river?

Work Problems

18. Daren and Cara painted a fence together in 8 hours. It would have taken Cara 12 hours to paint the fence by herself. What fraction of the fence did Cara paint? What fraction of the fence did Daren paint?

Progress Gradually

Percent Problems in Solving Equations - A

1. You are earning $10 an hour. You receive a 5% raise.
    a. Find the amount of your raise.
    b. Find your new hourly pay.

2. You are earning x dollars an hour. You receive a 5% raise.
    a. Express the amount of your raise in terms of x.
    b. Express your new hourly pay in terms of x.

3. An employee earns $21.00 per hour after receiving a 5% raise. What was the employee's hourly pay before the raise?

Percent Problems in Solving Equations - B

4. A store sold $4000 worth of merchandise in one day. The store is located in a state that requires a 6% sales tax on all items sold. How much money did the store collect in sales tax that day?

5. A store sold x dollars worth of merchandise in one day. The store is located in a state that requires a 6% sales tax on all items sold.
    a. Express in terms of x the amount of money the store collected in sales tax that day.
    b. Express in terms of x the total amount of money the store took in that day.

6. A store is located in a state that requires a 6% sales tax on all items sold. If the store brings in a total of $4240 in one day, how much of that total was sales tax?

Uniform Motion Problems

7. Two cyclists start at the same time from opposite ends of a course that is 63 miles long and ride towards one another. One cyclist is riding at a rate of 17 mph. The second cyclist is riding at a rate of 19 mph. After 1.5 h, how far has each traveled?

8. Two cyclists start at the same time from opposite ends of a course that is 63 miles long and ride towards one another. One cyclist is riding at a rate of 17 mph. The second cyclist is riding at a rate of 19 mph. After 2 h, will the cyclists have passed each other?

9. Two cyclists start at the same time from opposite ends of a course that is 63 miles long and ride towards one another. One cyclist is riding at a rate of 17 mph. The second cyclist is riding at a rate of 19 mph. After t hours, how far has each traveled?

10. Two cyclists start at the same time from opposite ends of a course that is 63 miles long and ride towards one another. One cyclist is riding at a rate of 17 mph. The second cyclist is riding at a rate of 19 mph. How long after they begin will they meet?

Investment Problems

11. You invest $5000 at an annual simple interest rate of 5.2%. A second investment, $1000 more than the first, is invested at an annual simple interest rate of 7.2%. After one year, how much interest has each investment earned?

12. You invest x dollars at an annual simple interest rate of 5.2%. A second investment, $1000 more than the first, is invested at an annual simple interest rate of 7.2%. After one year, how much interest has each investment earned?

13. You invest some money at an annual simple interest rate of 5.2%. A second investment, $1000 more than the first, is invested at an annual simple interest rate of 7.2%. After one year, the total annual interest earned is $320. How much is invested in each account?

Percent Mixture Problems

14. A chemist has a 200-milliliter solution that is 11% acid and a 500-milliliter solution that is 4% acid. If the chemist pours each solution into one beaker, how many milliliters of acid are in the beaker?

15. A chemist has x milliliters of a solution that is 11% acid and a 500-milliliter solution that is 4% acid. If the chemist pours each solution into one beaker, how many milliliters of acid are in the beaker?

16. A chemist has a solution that is 11% acid and 500 ml of a solution that is 4% acid. If the chemist pours each solution into one beaker, how many milliliters of the 11% acid solution should be used to make a 6% acid solution?

Explain the Meaning of an Answer

A chef is making 120 meatballs from 6 pounds of hamburger. Six pounds of hamburger contain 7200 calories.

1. Write a sentence explaining the meaning of the quotient of 120 and 6.

2. Write a sentence explaining the meaning of the quotient of 6 and 120.

3. Write a sentence explaining the meaning of the quotient of 7200 and 120.

4. Write a sentence explaining the meaning of the quotient of 7200 and 6.

5. There are 16 ounces in 1 pound. Which of the answers in Exercises 1 to 4 must you know to find the weight, in ounces, of 1 meatball?

6. An aerobic activity burns about 5 calories per minute. Which of the answers in Exercises 1 to 4 must you know to find how many minutes of aerobic activity are necessary to burn off the calories in one meatball?

7. If hamburger costs $3.50 per pound, which of the answers in Exercises 1 to 4 must you know to find the cost of one meatball?

8. Suppose the chef makes the same size meatballs using 8 pounds of hamburger instead of 6 pounds of hamburger. Which of the answers in Exercises 1 to 4 would you use to determine how many meatballs the chef makes?

Geometry

1. Which of the following cannot be used to form a triangle?
   a. Line segments whose lengths are 3 cm, 5 cm, and 7 cm
   b. Line segments whose lengths are 4 in., 5 in., and 10 in.
   c. Line segments whose lengths are 8.2 m, 6.45 m, and 11.2 m
   d. Line segments whose lengths are 7 yd, 2 yd, and 4 yd

2. Explain how to determine whether a triangle can be formed when the measures of three line segments are given.

3. If x, y, and z are the measures of the sides of a triangle, write a relationship between x, y, and z.

4. Take an 8.5 x 11 sheet of paper and roll it into a cylinder so that it has a height of 11 inches. Now roll the paper along the shorter edge so that it has a height of 8.5 inches. Which configuration holds more popcorn?

5. Suppose a manhole cover is a square with dimensions 3 feet by 3 feet, and it rests on a square opening that is 2' 10" by 2' 10". Is there a way for the manhole cover to fall through the hole?

6. Suppose a manhole cover is a circle with radius 2 feet, and it rests on a circular opening that has a radius of 1' 10". Is there a way for the manhole cover to fall through the hole?

7. What is the shape of the nut on top of a fire hydrant? Why is it shaped this way?

8. A slightly different way to get at measurement concepts:
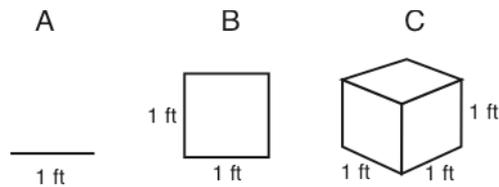
1. Which of the figures could be used to measure the amount of carpet needed for a room?
2. Which of the figures could be used to measure the height of a person?
3. Which of the figures could be used to measure the amount of sand in a sandbox?

Variable Expressions

For each of the problems below, write an algebraic expression that answers the question.

1. A candy manufacturer has x pieces of chocolate. If each piece of chocolate weighs y pounds, how many pounds of chocolate does the manufacturer have?

2. A candy manufacturer has x pounds of chocolate and purchases another y pounds of chocolate. How many pounds of chocolate does the manufacturer have?

3. A candy manufacturer has x pounds of chocolate. To make a piece of candy requires y pounds of chocolate. How many pieces of candy can the manufacturer make?

4. A candy manufacturer had x pounds of chocolate and used y pounds of chocolate to make bonbons. How many pounds of chocolate does the manufacturer have after making the bonbons?

5. A candy manufacturer has x pounds of chocolate and y pounds of caramel. What percent of the total is chocolate?

Application problems which require students to identify the quantity to be found and to assign a variable to an unknown quantity come in two forms: (1) the variable represents the quantity to be found, and (2) the variable does not represent the quantity to be found. Exercise 1 below is of the first type. Exercises 2 and 3 are of the second type, with Exercise 3 being the more difficult exercise for students.

1. The width of a rectangle is 15 feet. If the perimeter of the rectangle is 80 feet, what is the length of the rectangle?

2. The length of a rectangle is 3 feet more than twice the width. If the perimeter of the rectangle is 80 feet, what is the length of the rectangle?

3. Kevin begins a course riding at an average speed of 6 mph. One hour later, Michelle travels the same course at an average speed of 12 mph. How far from the starting point does Michelle overtake Kevin?

Introducing the Need for Two Variables

One way to introduce equations in two variables is to give students a situation that requires two variables. Here are some suggestions.

1. Hana and Miya take turns driving on a 45-hour drive across country. How long did Miya drive?

2. A grocer has a 10-pound mixture of walnuts and pecans. How many pounds of each are in the mixture?

3. The sum of two numbers is 10. What are the numbers?

4. A 15-ounce gold alloy is made from silver and gold. How many ounces of each were used?

The Effect of Language on Translating Equations in Two Variables

Here are some problems that require students to translate a two-variable situation into an equation. Although the odd-numbered problems and even-numbered problems are exactly the same, students have more difficulty with the wording in the odd-numbered problems.

1. At this college, there are six times as many students as professors.

2. At this college, the number of students is six times the number of professors.

3. There are ten times as many red M&M's as blue M&M's.

4. The number of red M&M's is ten times the number of blue M&M's.

5. Hana drove five times as many miles as did Miya.

6. The number of miles driven by Hana was five times the number of miles driven by Miya.

Slope

Preparing to Graph Lines

Place a dot at the point $P(-3, -4)$. Move to the right one unit and up two units and place another dot. At the new point, again move to the right one unit and up two units and place another dot. Do this 3 more times. Describe the shape of a smooth graph drawn through the dots.

Application of Slope

Average speed is distance traveled divided by the time of travel. Use this for the following exercises.

1. A hiker walked 6 miles in 2 hours.
    a. What is the average walking speed of the hiker?
    b. Write a sentence that explains what you did in part **a**.

2. After hiking for one hour, a hiker was 2 miles from the starting point of a hiking trail. Two hours later, the hiker was 8 miles from the starting point.
    a. What was the average walking speed of the hiker for the last two hours?
    b. Write an explanation of how you found the answer to part **a.**

Average miles per gallon is calculated by dividing the number of miles driven by the number of gallons of gasoline consumed.

3. A car went 200 miles and used 10 gallons of gas.
    a. What is the average miles per gallon for that car?
    b. Write a sentence that explains what you did in part **a**.

4. During the first part of a trip, a car used 5 gallons of gas to travel 100 miles. After 200 miles, the car had used 9 gallons of gas.
    a. What was the average miles per gallon for the second part of the trip?
    b. Write a sentence that explains what you did in part **a**.

Average cost is calculated by dividing total cost of all items by the number of items purchased.

5. A merchant purchased 200 reams of paper for $400.
    a. What was the average cost of a ream of paper?
    b. Write a sentence that explains what you did in part **a**.

6. The cost of 200 reams of paper is $400. Because of a volume discount, a merchant purchased 250 reams for $487.50.
    a. What is the average cost of each ream of paper over 200 reams purchased by the merchant?
    b. Write a sentence that explains what you did in part **a**.

Polynomials

1. To which of the following does the rule $(x^m y^n)^p = x^{mp} y^{np}$ apply?

   a. $(2x)^3$

   b. $(xy^2 z^3)^4$

   c. $(2 + x^2)^3$

   d. $(x - y)^3$

   e. $(-4ab^4 c^2)^5$

2. Is it possible to add two polynomials, each of degree 3, and have the sum be a polynomial of degree 2?

3. Is it possible to add two polynomials, each of degree 3, and have the sum be a polynomial of degree 4?

4. Is it possible to multiply two polynomials, each of degree 3, and have the product be a polynomial of degree 6?

5. Two linear factors of $x^4 + x^3 - 7x^2 - x + 6$ are $x - 1$ and $x + 3$. Explain how to find the other two linear factors of $x^4 + x^3 - 7x^2 - x + 6$.

Quadratic Equations

6. What are the two solutions of $x + 3 = \pm\frac{2}{3}$?

7. One solution of the equation $2x^2 - 5x + c = 0$ is 4. What is the other solution?

Projects

Percent discount

A retailer is having a sale on items already on sale.

1.   Suppose a retailer has marked down an item 30% but is offering an additional 20% off the already reduced items. Does this amount to a 50%-off sale?

2.   Would it be better to get one discount of 50% or a 30% discount followed by a 20% discount?

3.   Suppose the retailer marked down an item 20% and then offered 30% off on the already marked down item. Is that the same as marking down the item 30% and then 20% as in part 1?

4.   Does a 40% discount followed by a 10% discount result in an item being less expensive than a 30% discount followed by a 20% discount?

5.   If two discounts whose percents sum to 50% are given, how should they be allocated to create the least expensive item?

6.   Suppose the retailer marked down an item 70% but is offering an additional 30% off the already reduced item. Does this amount to a 100%-off sale?

7.   Suppose the retailer marked down an item 30% but is offering an additional 70% off the already reduced item. Is this the same as the result in part 6?

Percent Increase and Decrease

During the bear market of 2008, the value of the S&P decreased approximately 40%. Bull markets, where the value of the S&P increases, have usually followed bear markets. Historically, the percent increases for the first four years of a bull market have been 38%, 11%, 5%, and 4%. Suppose an investor began with a $100,000 investment at the beginning of 2008 and lost 40% of that investment during 2008. Assuming the investor just left the investment (did not add money to or take money out of the investment) and that the typical, historical returns are expected, in what year would the investment regain its value of $100,000?

Population and Land Allocation

In this project you are asked to determine hypothetical land allocation for the world's population today. Use the figure 6×10^9 for the current world population and the figure 3.0×10^8 for the current U.S. population. One square mile is approximately $2.8 \times 10^7 \text{ft}^2$.

1.   If every person in the world moved to Texas and each person were given an equal amount of land, how many square feet of land would each person have? The area of Texas is $2.619 \times 10^5 \text{ mi}^2$.

2. If every person in the United States moved to Rhode Island and each person were given an equal amount of land, how many square feet of land would each person have? The area of Rhode Island is 1.0×10^3 mi^2. Round to the nearest whole number.

3. Suppose every person in the world were given a plot of land the size of a two-car garage (22 ft x 22 ft).
   a. How many people would fit in a square mile? Round to the nearest hundred.
   b. How many square miles would be required to accommodate the entire world population? Round to the nearest hundred.

4. If the total land area of Earth were divided equally, how many acres of land would each person be allocated? Use a figure of 5.7×10^7 mi^2 for the land area of Earth. One acre is 43,560 ft^2. Round to the nearest tenth.

5. If every person on Earth were given a plot of land the size of a two-car garage, what would be the carrying capacity of Earth? Round to the nearest hundred billion.

6. a. The IPv6 Internet protocol is a 32-digit hexadecimal number. How many IPv6 addresses are possible? b. How many IPv6 addresses are possible for every person on Earth?

Magnitude of Numbers

This activity allows students to examine the magnitude of 1 billion and to use conversion among units.

1. Measure your heart rate in beats per minute.

2. Assuming your heart rate is constant, how many minutes would you have to live to have one billion heartbeats?

3. How old in months and years are you when you have your one-billionth heartbeat? Assume all years have 365 days.

4. The national debt is increasing about $3 billion per day. How old would you be on your three-billionth heartbeat?

5. The U.S. government recently provided $700 billion to financial institutions. How long would you have to live to have 700 billion heart beats?