PROJECTS APPLYING LINEAR

ALGEBRA TO CALCULUS

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MAJOR PROBLEMS IN TEACHING A LINEAR ALGEBRA CLASS:

- Having enough time in the semester to cover the syllabus.

- Some problems are long and involved.
  - Solving Systems of Equations.
  - Finding Eigenvalues and Eigenvectors

- Concepts can be difficult to visualize.
  - Rotation and Translation of Vectors in multi-dimensioanl space.
  - Solutions to systems of equations and systems of differential equations.

- Some problems may be repetitive.

A SOLUTION TO THESE ISSUES: OUT OF CLASS PROJECTS.

THE BENEFITS:

- By having the students learn some material outside of class, you have more time in class to cover additional material that you may not get to otherwise.

- Students tend to get lost when the professor does a long computation on the board. Students are like "scribes."

- Using technology, students can more easily visualize concepts.

- Doing the more repetitive problems outside of class cuts down on boredom inside of class.

- Gives students another method of learning as an alternative to be lectured to.

APPLICATIONS OF SYSTEMS OF LINEAR EQUATIONS: (taken from Lay, fifth edition)

- Nutrition:

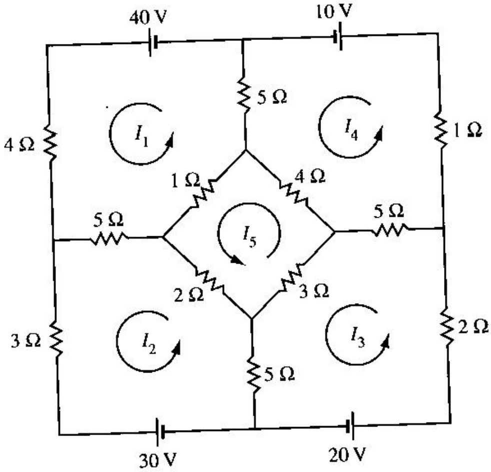
4. A dietician is planning a meal that supplies certain quantities of vitamin C, calcium, and magnesium. Three foods will be used, their quantities measured in appropriate units. The nutrients supplied by these foods and the dietary requirements are given here.

	Milligrams (mg) of Nutrients per Unit of Food			Total Nutrients Required (mg)
Nutrient	Food 1	Food 2	Food 3	
Vitamin C	10	20	20	100
Calcium	50	40	10	300
Magnesium	30	10	40	200

Write a vector equation for this problem. State what the variables represent, and then solve the equation.

- Electrical Currents:

8.



- Population Distribution:

12. [M] Budget Rent A Car in Wichita, Kansas, has a fleet of about 450 cars, at three locations. A car rented at one location may be returned to any of the three locations. The various fractions of cars returned to each location are shown in the matrix below. Suppose that on Monday, there are 304 cars at the airport (or rented from there), 48 cars at the east side office, and 98 cars at the west side office. What will be the approximate distribution of cars on Wednesday?

Cars Rented From:

Airport	East	West	Returned To:
.97	.05	.10	Airport
.00	.90	.05	East
.03	.05	.85	West

VISUALIZING LINEAR TRANSFORMATIONS:

in each of the five exercises below, do the following:

(a) Find the matrix A that performs the desired transformation. (Note: In Exercises 4 and 5 where there are two transformations, multiply the two matrices with the first transformation matrix being on the right.)

(b) Let $T : \Re^2 \to \Re^2$ be the transformation $T(x) = Ax$ where A is the matrix you found in part (a). Find $T(2,3)$.

(c) Using MAPLE, sketch the vector $[2,3]$ and the vector you found in part (b) on the same set of axes. Sketch $[2,3]$ in red and the vector from part (b) in blue.

EXERCISES:

1.) Rotates a vector counterclockwise (about the origin) through $\pi/3$ radians (60 degrees). Round all numbers to two decimal places.

2.) Simultaneously contracts a vector horizontally by a factor of $1/2$ and expands a vector vertically by a factor of 2.

3.) Simultaneously does a vertical shear transformation mapping $[1, 0]$ to $[1, -2]$, and a horizontal shear mapping $[0, 1]$ to $[3, 1]$.

4.) First reflects points through the horizontal x_1-axis and then reflects points through the line $x_2 = -x_1$.

5.) First performs a horizontal shear that transforms $[0, 1]$ into $[-2, 1]$ (leaving $[1, 0]$ unchanged) and then refelcts points through the line $x_2 = x_1$.

LINEAR TRANSFORMATIONS AND CALCULUS:

1.) a) Prove that $T : C^1 \to C^1$ defined by $T(f) = f'$ for all functions $f \in C^1$ is a linear transformation.

b) Find $T(x^2 + 5x + 3)$

2.) a) Prove that $T : C_{0,1} \to \Re$ defined by $T(f) = \int_0^1 f(x)\,dx$ is a linear transformation.

b) Find $T(x^2 + 5x + 3)$

3.) a.) Prove that $T : C^2 \to C^2$ defined by $T(y) = y'' - 4y$ for all functions $y \in C^2$ is a linear transformation.

b.) Show that e^{2x} are e^{-2x} are both in $Ker(T)$.

c.) Consider the differential equation $y'' - 4y = \sin x$. Show that $y = -\frac{1}{5}\sin x$ is a particular solution to this differential equation.

d.) Suppose that $\{\, e^{2x}, e^{-2x}\}$ forms a basis for $Ker(T)$. Use this and part (c) to find the solution *set* to the differential equation $y'' - 4y = \sin x$.

SYSTEMS OF DIFFERENTIAL EQUATIONS:

- Using eigenvalues and eigenvectors to solve systems of differential equations.
- Using systems of equations to solve initial value problems.
- Graphing phase portraits and specific solutions to initial value problems.

$$\begin{aligned} x_1'(t) &= 2x_1 + 2x_2 \\ x_2'(t) &= x_1 + 3x_2 \end{aligned}$$

The coefficient matrix is

$$A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$$

Step 1: Determine the eigenvalues and corresponding eigenvectors:

Obeserve that the eigenvalues are $\lambda_1 = 1$ and $\lambda_2 = 4$ with corresponding eigenvectors

$$\mathbf{v_1} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \text{ and } \mathbf{v_2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Step 2: In vector form, the solution to the system of differential equations is

$$\mathbf{x} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} k_1 e^t + \begin{bmatrix} 1 \\ 1 \end{bmatrix} k_2 e^{4t}$$

Step 3: Obtain the final answers for $x_1(t)$ and $x_2(t)$ by reading across:

$$\begin{aligned} x_1(t) &= 2k_1 e^t + k_2 e^{4t} \\ x_2(t) &= -k_1 e^t + k_2 e^{4t} \end{aligned}$$

Check your work: It's always a good idea to check your work.

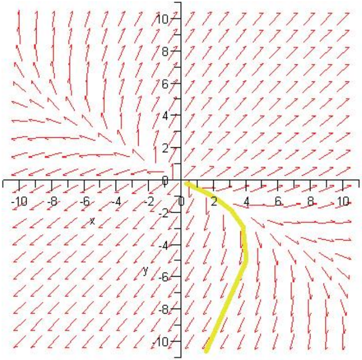
Observe $x_1'(t) = 2k_1 e^t + 4k_2 e^t$ which equals $2x_1 + 2x_2$ and $x_2'(t) = -k_1 e^t + 4k_2 e^{4t}$ which equals $x_1 + 3x_2$.

Getting specific solutions: Suppose we wanted to solve the system on the previous page subject to the initial conditions $x_1(0) = 4$ and $x_2(0) = -5$. Then using our solution from Step 3 and plugging in 0 for t (recall $e^0 = 1$) we obtain

$$\begin{aligned} 4 &= 2k_1 + k_2 \\ -5 &= -k_1 + k_2 \end{aligned}$$

Solving for k_1 and k_2 we obtain $k_1 = 3$ and $k_2 = -2$.

The Phase Portrait:



Observe the point $(4, -5)$ is highlighted since that corresponds to $t = 0$.