ACTIVE LEARNING THROUGH FILL-IN-THE-BLANK PROOFS

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Background in Proofs course

- Liberal arts college with 1600 students
- 17 students Fall 2017 (more commonly 8-12)
- Math majors (and occasional minors)
- *A Transition to Advanced Mathematics, 8th edition,*

  by D. Smith, M. Eggen, and R. St. Andre

Basic Format of Class

- Weekly Quizzes (60%)
- Final Exam (20%)
- Homework (20%)
  - Previous material: complete proofs/problems
  - New material: Answer reading questions and complete easy problems related to reading
  - All work (photos or text) uploaded to Moodle LMS

The Problem

- When students are put in groups to discuss how to write a proof,

  either

  - one student shows the others how he/she completed the proof,

  or

  - all students have no idea where to start.

How can I facilitate learning without lecturing?

Example Fill-in-the-blank proof

- Prove that, for every rational number z and every irrational number x, there exists a unique irrational number such that $x + y = z$.

- **Proof:** Let z be an arbitrary rational number and let x be an arbitrary irrational number. (We first need to show that there exists a number y such that $x + y = z$.)
- Set $y =$ ________________.

  $z - x$

- Then $x + y = x +$__________ $= z$.

  $z - x$

Proof, continued

- This proves there exists a number y such that $x + y = z$.

- (Now we need to verify that y is irrational.)

Proof, continued

- Using a proof by contradiction, suppose y is rational.
- Since y is rational, by definition, we can write

  $y =$ ____________ where ________________

  p/q $\quad$ $p, q \in \mathbb{Z}, q \neq 0.$
- Recalling that z is rational from ____________,

  the hypothesis

by definition, we can write $z =$ ____________

r/s where $\mathrm{r}, s \in \mathbb{Z}, s \neq 0.$

Proof, continued

- Thus, $x = \mathrm{z} - \mathrm{y} =$ ______________________________

  $\frac{r}{s} - \frac{p}{q} = \frac{rq-ps}{sq}$.

- Since ________________ and ___________ are integers

  $rq - ps$ sq

- and ________ $\neq 0$, then x is ___________________.

  sq rational

- However, from the hypothesis, x is _________________.

  irrational.

Proof, continued

- Thus, our assumption that ___________ is false, and

  y is rational

  we conclude that y is irrational.

  (Now show uniqueness.)

Implementation

- How to implement?

  Write the proof and then delete information.

Examples:

- **To prove a relation R on A is transitive,**

  let x, y and $z \in A$ and assume _______ and _______.

  We need to show _________.

- $x \in B \cup \bigcap_{\alpha \in \Delta} A_\alpha$ iff $x \in \mathrm{B}$ or $x \in \bigcap_{\alpha \in \Delta} A_\alpha$

  by the definition of ______

More implementation

More Examples:

- **Via Theorem 4.1.1**, we need to show that

  (i) $Dom(f^\circ f^{-1}) =$ ______________________

  (ii) $\forall\, x \in Dom(f \circ f^{-1}),\quad (f \circ f^{-1})(x) =$ ____

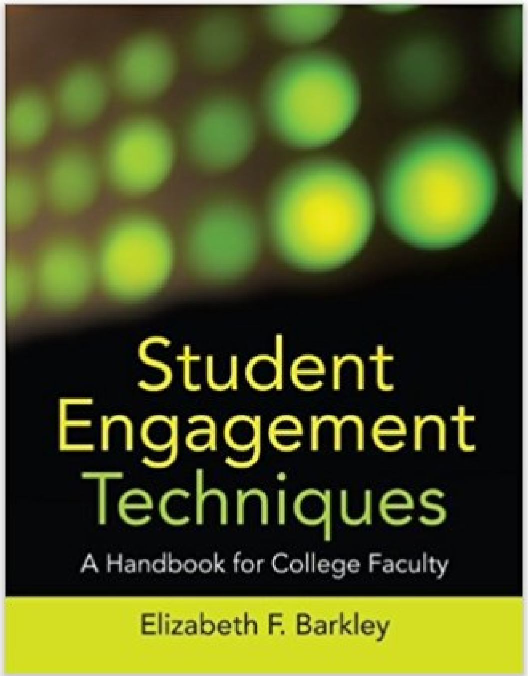
- $f^{-1}(b) = (I_A \circ f^{-1})(b)$

  by **Theorem** _______ **(applied to $\boldsymbol{f^{-1}}$)**

Resource

Student Engagement Technique (SET)
Frames, pp.191-194

"…templates […] make explicit
the deep structural thinking
that underlies good academic argument,
and
help students by providing
them with tools that guide them into
thinking clearly and critically
in a direct and immediate way. "



Jossey-Bass (2010)

Positives

1. Students work together (and continue to work together outside of class)
2. Discussion (and disagreement) happens.
3. Writing and reasoning improved. (More than having students submit and resubmit proofs)
4. Students talk to me instead of resorting to online resources like Chegg.
5. I'm not lecturing.

Need to work on

Proofs can be completed using different tools…

Students don't always fill in what I expect them to…

Providing enough information to complete a proof but not so much as to turn off thinking

Want more info?

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Questions?